## Laboratory experiment

# Study of laws of collision on an air-cushion track

## 1.1 Task

The aim of this experiment is to study elastic and inelastic collisions of bodies moving without friction on a horizontal air-cushion track. For different mass ratios of the bodies (gliders), determine their kinetic energies and momenta before and after the collision.

## 1.2 Theory

### 1.2.1 Collisions of bodies

Under the influence of the applied forces, bodies move smoothly in accordance with the laws of motion. If two or more bodies interfere with each other in their motion, they collide, i.e., the magnitudes and directions of motion of these bodies change rapidly.

The concept of collision is very general. For example, we can talk about collision of cars, galaxies, elementary particles, and it is clear that the processes and mechanisms of these collisions are quite different and different processes are involved.

The collision of rigid and elastic bodies is also called the impact of bodies. During a very short impact, tremendous impact forces are generated at the point of contact of the bodies, which cause an abrupt change in their motion, can cause their deformation or even breaking. Because of the large magnitudes of the impact forces, we can usually neglect the action of other forces during the impact.

If the law of conservation of kinetic energy applies in the collision of bodies, we speak of an *elastic (perfectly elastic) collision*, if not, we speak of an *inelastic collision*. If after the collision there is no rebound of the bodies and they remain connected, we speak of a *perfectly inelastic collision*.

### 1.2.2 Conservation laws

If we do not know the exact mechanism of the collision of bodies (their mutual force interaction), we cannot predict their outcome unambiguously. It depends on the shape of the bodies, their elasticity, surface roughness, etc. The mutual forces that the bodies exert on each other during a collision are internal forces, forces of action and reaction. Since the action of external forces can be neglected during the collision, the colliding bodies form an isolated reference frame and the law of conservation of momentum applies to them. Thus, even without knowing anything more about the collision mechanism, for the collision of two bodies, we can write the law of conservation of momentum

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_1' + m_2\mathbf{v}_2', \tag{1.1}$$

thus the total momentum of the two bodies before the collision is equal to their total momentum after the collision. Here $m_i$ are the masses of the bodies, $\mathbf{v}_i$ are their velocities before the collision, and $\mathbf{v}_i'$ are their velocities after the collision ($i = 1, 2$).

In addition, in an elastic collision, the law of conservation of kinetic energy also applies, which has the form

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2, \tag{1.2}$$

thus the total kinetic energy of the bodies before the collision is equal to the total kinetic energy of the bodies after the collision. Here $v_i^2 = \mathbf{v}_i \cdot \mathbf{v}_i$ is the square of the magnitude of the velocity of the bodies.

Equations (1.1) and (1.2) form a set of four equations for six unknown components of vectors $\mathbf{v}_1', \mathbf{v}_2'$ after the collision and thus they do not describe the general elastic collision uniquely. Even in the case of the plane problem, the number of equations is not sufficient. Only in the case of a one-dimensional (linear) problem (the velocities before and after the collision lie on the same straight line–we speak of a linear collision) we have two equations for two unknowns, and thus we can describe the elastic collision uniquely, regardless of the collision mechanism.

In the following, we will discuss the perfectly elastic and perfectly inelastic linear collision of two bodies, where one of them (the target body) will always be at rest before the collision.

### 1.2.3 Linear elastic collision

The laws of conservation of momentum and energy in this particular case have the form

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2', \tag{1.3a}$$

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2, \tag{1.3b}$$

where $v_1, v_2, v_1'$, and $v_2'$ are the oriented velocity magnitudes, i.e., the respective components of the velocity vectors in the direction of the line along which the impact takes place. Since we consider the second body prior to the collision at rest, we can write $v_2 = 0$, $v_1 = v$ and introduce the quantity $\mu = m_1/m_2$. The equations (1.3) then take the form

$$\mu v = \mu v_1' + v_2', \tag{1.4a}$$

$$\mu v^2 = \mu v_1'^2 + v_2'^2 \tag{1.4b}$$

From Eq. (1.4a) we can isolate the velocity $v_2' = \mu(v - v_1')$ and after substitution into Eq. (1.4b) we obtain

$$\mu(v^2 - v_1'^2) = \mu^2(v - v_1')^2 \quad \Rightarrow \quad \mu(v - v_1')(v + v_1') = \mu^2(v - v_1')(v - v_1'). \tag{1.5}$$

One of the solutions of Eq. (1.5) is $v_1' = v$, after substituting it into Eq. (1.4a) we get $v_2' = 0$, where we see that this solution describes the situation before the collision, for this reason it is not interesting for us and we will not deal with it further. After dividing Eq. (1.5) by $\mu(v - v_1') \neq 0$ we get

$$v_1' = \frac{\mu - 1}{\mu + 1}v \tag{1.6}$$

and after substitution into Eq. (1.4a)

$$v_2' = \frac{2\mu}{\mu + 1} v \tag{1.7}$$

Depending on the value of the ratio $\mu = m_1/m_2$, from Eqs. (1.6) and (1.7) these conclusions follow:

- If $\mu > 1, (m_1 > m_2)$, then it holds[1]: $\mathrm{sign}(v_1') = \mathrm{sign}(v_2') = \mathrm{sign}(v)$, so both bodies after the collision will move in the same direction as the first body before the collision. If $\mu \gg 1, (m_1 \gg m_2)$, after the collision $v_1' \approx v$ (the first body will continue with the same velocity), and $v_2' \approx 2v$ (the second body will be accelerated to twice the velocity of the first one).

- If $\mu = 1, (m_1 = m_2)$, then it holds: $v_1' = 0$, $v_2' = v$, i.e., after the collision the first body will stop and the second body will move with the same velocity as the first body before the collision (the bodies will exchange their momentum).

- If $\mu < 1, (m_1 < m_2)$, then it holds: $-\mathrm{sign}(v_1') = \mathrm{sign}(v_2') = \mathrm{sign}(v)$, i.e., the first body after the collision will change its direction (it bounces back), the second body after the collision will move in the same direction as the first body before the collision. If $\mu \approx 0, (m_1 \ll m_2)$, then $v_1' \approx -v$, $v_2 \approx 0$, i.e., the first body will bounce back with the same velocity and the second body will remain at rest.

For the momenta of the bodies after the collision, from Eqs. (1.6) and (1.7) we obtain

$$p_1' = \frac{\mu - 1}{\mu + 1} p, \qquad p_2' = \frac{2}{\mu + 1} p, \tag{1.8}$$

for the kinetic energies it holds

$$T_1' = \left( \frac{\mu - 1}{\mu + 1} \right)^2 T, \qquad T_2' = \frac{4\mu}{(\mu + 1)^2} T. \tag{1.9}$$

Of course, it also holds $p_1' + p_2' = p$ and $T_1' + T_2' = T$.

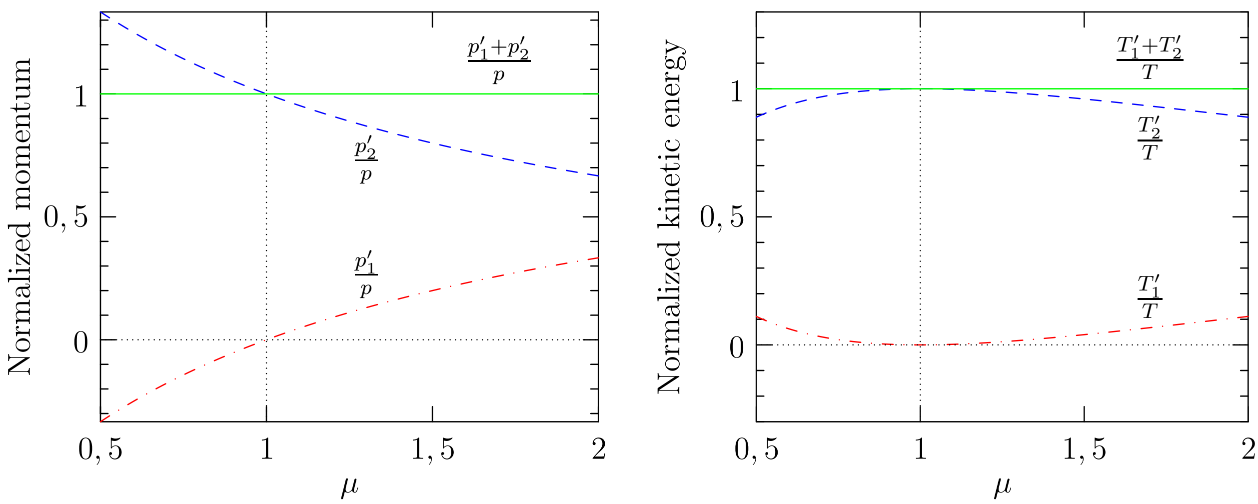


Figure 1.1: Normalized momenta and kinetic energies of individual bodies after elastic collision for different values of the parameter $\mu$.

[1]The function sign() is defined as follows: $\mathrm{sign}(x) \equiv 1$ for $x \geq 0$, $\mathrm{sign}(x) \equiv -1$ for $x < 0$.

Next, we can find out under what conditions (for what ratio of masses for a given $T$) the first body will transfer the greatest amount of energy to the second one by elastic collision. Taking the derivative of the coefficient in the right equation of Eqs. (1.9) gives[2]

$$\frac{\mathrm{d}}{\mathrm{d}\mu}\left[\frac{4\mu}{(\mu+1)^2}\right] = -\frac{4(\mu-1)}{(\mu+1)^3} \stackrel{!}{=} 0 \qquad \Rightarrow \qquad \mu = 1. \tag{1.10}$$

So the second body gets the maximum kinetic energy if the masses of both the bodies are equal ($m_1 = m_2$), and it holds $T_2' = T$, $T_1' = 0$.

### 1.2.4 Linear perfectly inelastic collision

Next, we will consider the case where the bodies collide inelastically in such a way that they stick together and thus $v_1' = v_2' = v'$. In this case, we cannot use the law of conservation of kinetic energy to describe the collision, since part of the energy is dissipated in plastic deformation and possibly heating of the bodies. However, the law of conservation of momentum is valid and we can write (again assuming that the second body is at rest before the collision, so $v_2 = 0$, and again denoting $v_1 = v$)

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' \quad \Rightarrow \quad m_1v = (m_1+m_2)v' \quad \Rightarrow \quad v' = \frac{\mu}{\mu+1}v. \tag{1.11}$$

For the momentum of the (joined) bodies after the collision we get

$$p_1' = \frac{\mu}{\mu+1}p, \qquad p_2' = \frac{1}{\mu+1}p, \tag{1.12}$$

and again, it holds that $p_1' + p_2' = p$. For the kinetic energy of the bodies after the collision it holds

$$T_1' = \left(\frac{\mu}{\mu+1}\right)^2 T, \qquad T_2' = \frac{\mu}{(\mu+1)^2}T. \tag{1.13}$$

In an inelastic collision, part of the kinetic energy is dissipated in plastic deformation, so $T_1' + T_2' \neq T$. We can calculate the magnitude of this energy (deformation work) as

$$\Delta T = T - (T_1' + T_2') = \frac{1}{\mu+1}T. \tag{1.14}$$

From here, it can be seen that:

- If $m_1 \gg m_2$ ($\mu \gg 1$), then $\Delta T \approx 0$, i.e., only a minimum of kinetic energy is dissipated in the inelastic collision.
- If $m_1 = m_2$ ($\mu = 1$), then $\Delta T = T/2$, and thus a half of the kinetic energy is dissipated in the inelastic collision.
- If $m_1 \ll m_2$ ($\mu \approx 0$), then $\Delta T \approx T$, i.e., almost all the kinetic energy is dissipated in the inelastic collision.

[2] For $\mu < 1$ the derivative is positive, the function is then increasing; for $\mu > 1$ the derivative is negative, the function is decreasing. From here, it follows that this extreme represents the maximum.

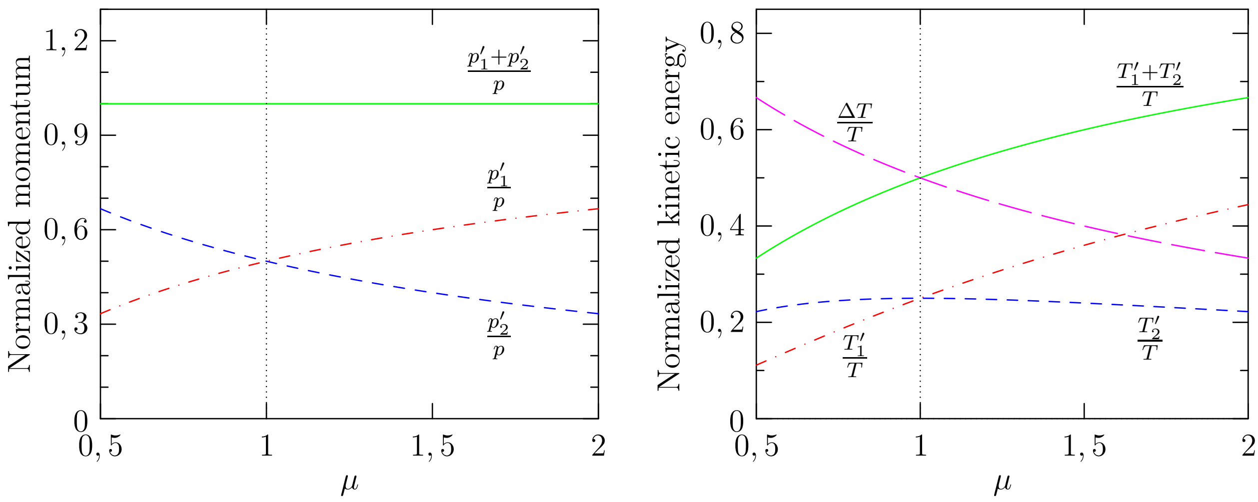


Figure 1.2: Normalized momenta and kinetic energies of individual bodies after perfectly inelastic collision for different values of the parameter $\mu$.

## 1.3 Experiment

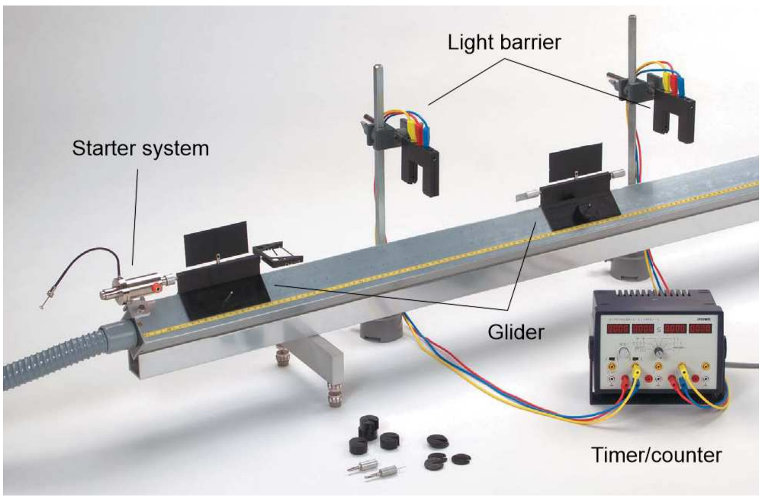


Figure 1.3: Air-cushion track for the study of collision of bodies.

The aim of this experiment is to verify the validity of the relations for the redistribution of the momentum and kinetic energy of bodies in relation to the ratio of their masses $\mu = m_1/m_2$ in a linear elastic collision and a linear perfectly inelastic collision [relations (1.8), (1.9), (1.12), and (1.13)].

The above formulas were derived under the assumption that during a very short collision we can neglect all external forces and consider the colliding bodies an isolated reference frame. In order to be able to measure the velocities of the bodies before and after the collision, the experiment is placed on a horizontal air-cushion track, see Fig 1.3. The air-cushion track is placed horizontally, so that the gravitational force does not act in the direction of motion of the bodies (gliders). They move almost without friction on the air cushion, so their motion before and after collision is uniform (they move at constant velocities).

### 1.3.1 Connecting and setting-up the experiment

Check that the starter system at the end of the air-cushion track is turned with the movable end towards the track. If not, loosen the clamping screws and rotate it.

Connect the starter system to the timer terminals, see Fig. 1.4, marked **START**, with the red terminal on the starter system to the yellow terminal on the timer. Connect the light barriers to the timer terminals **1** and **3** so that the corresponding colours match each other. Connect the barrier closer to the starter to the terminals **1**. Switch the timer mode rotary switch to the position **3**, see

Fig. 1.4.

Switch both slide switches to the rightmost position. In this mode and wiring, the first display shows the duration of the first shading of the first light barrier and the second display the duration of the second shading of the first light barrier. The third and fourth displays function similarly for the second light barrier. The measured times are reset by pressing the **RESET** button.

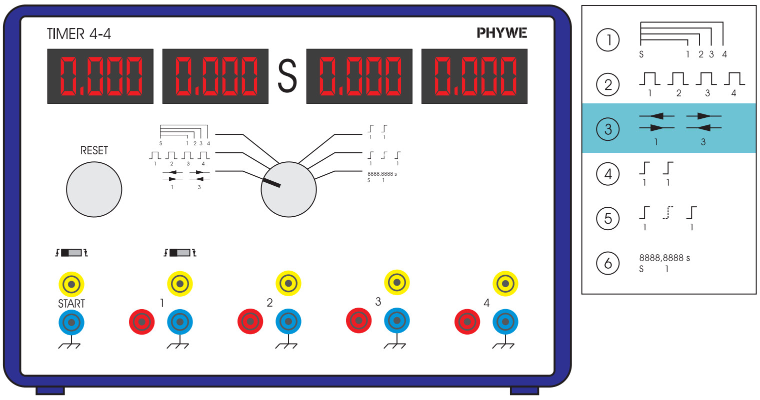


Figure 1.4: Timer.

Switch on the blower and set the desired airflow[3] (e.g. position 3-4). Check that the air track is horizontal (gliders do not move on it spontaneously when set at rest), if not, adjust it to the best possible horizontal position using the appropriate screws. Position the light barriers so that they measure the time before and after the collision when they are shaded by the freely moving gliders.

### 1.3.2 Procedure

**Elastic collisions**

1. Connect and set up the experiment according to the procedure described in the previous paragraph.

2. Attach the fork with a rubber band to one of the gliders and the plate with plug to the other to provide an elastic collision. Attach the $l = 10$ cm screens to both the gliders. On the glider that will be the launcher, place **symmetrically** two weights with masses 50 g.

3. Use the portable balance to measure the masses of the gliders ($m_1$ and $m_2$).

4. Check whether the gliders can move freely on the track. Set the starter device's launcher to **the middle latch position**, use the magnet to attach the accelerated glider to it. Place the second glider between the light barriers and bring it to rest.

5. Press the **RESET** button to reset the timer and launch the glider. Calculate the speeds of each glider as $v_i = l/\Delta t_i$, where $\Delta t_i$ are the shading durations of the individual light barriers (shown on the individual displays) and $l$ is the length of the screens on the gliders. Conduct the measurements at least 5×.

6. Add symmetrically two 10 g weights to the target glider and continue the measurement with point 4.

7. Plot a graph showing the theoretical momentum dependences [relations (1.8)] and their sum, together with the measured values, as a function of the mass ratio $\mu$.

8. Plot a graph showing the theoretical kinetic energy dependences [relations (1.9)) ] and their sum, together with the measured values, as a function of the mass ratio $\mu$.

[3] If the airflow is too low, the gliders would not move on the air cushion.

**Perfectly inelastic collisions**

The measurement is carried out in the same way as in the case of elastic collisions, only the fork with the rubber band and the plug with plate are replaced by the plug with needle and the plug with tube filled with wax ensuring the connection of the two gliders after the collision. With prolonged use, the needle will puncture the wax filling (you will hear the impact when the collision occurs). In this case, just push the wax filling into the tube.

- Plot a graph showing the theoretical momentum dependences [relations (1.12)] and their sum together with the measured values as a function of the mass ratio $\mu$.
- Plot a graph showing the theoretical kinetic energies [relations (1.13)], their sum, and deformation work [relation (1.14)], together with the measured values, as a function of the mass ratio $\mu$.

## 1.4 References

More on collisions can be found e.g. in textbook:

Jiří Bajer: Mechanika 2, *Univerzita Palackého v Olomouci*, Olomouc, 2004.

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