Laboratory experiment

Measurement of Doppler effect

1.1 Task

Measure the frequency shift of the ultrasonic wave if the observer (receiver) or source (transmitter) of this wave move to each other. Plot a graph of the dependence (change) of the frequency as a function of the speed of the source or receiver. Compare the measured data with the theoretical values.

1.2 Doppler effect

If the acoustic wave source or receiver move relative to each other, the frequency of the wave detected by the receiver changes against the frequency that he would detect if the receiver or source did not move relative to each other. This effect is called Doppler effect because it was discovered in 1842 by Christian Doppler[1], and it applies to all known types of waves, not only to mechanical ones, but also for electromagnetic waves (light, radio waves, ...). Here, we will only focus ourselves on Doppler effect for acoustic waves propagating in fluids, namely, the air.

If the acoustic wave source moves with respect to air, the center of the emitted waves shifts, so that the wavefront is compressed in front of the source and extended behind it, see Fig. 1.1.

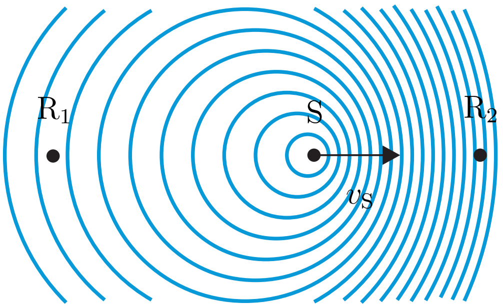

Figure 1.1: Moving source.

The source S approaches the receiver R$_2$ with speed v_S, which means that during every period T it travels the distance $v_S T$. As a result, the wave crests in front of the source are not separated by the distance of the wavelength $\lambda = cT$, where c is the sound speed, but, by the distance of $\lambda' = \lambda - v_S T$, which is the wavelength registered by the receiver. This wavelength is related to frequency

$$f' = \frac{c}{\lambda'} = \frac{c}{\lambda - v_S T} = \frac{c}{cT - v_S T} = \frac{c}{c - v_S} f, \tag{1.1}$$

where $f = 1/T$ is the frequency emitted by the source. If $v_S < c$, then $f' > f$, the receiver registers a higher frequency.

If the source recedes from the receiver R$_1$ with speed v_S, the distance between the adjacent wave crests is enlarged by the distance of $v_S T$, so the registered wavelength $\lambda' = \lambda + v_S T$, which is related to frequency

$$f' = \frac{c}{c + v_S} f. \tag{1.2}$$

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[1] Christian Johann Doppler (1803–1853), Austrian mathematician and physicist, from 1837 to 1847 he worked as professor of elementary mathematics and practical geometry at the Estates Engineering School in Prague.

The receiver from which the source recedes thus registers a lower frequency.

From Eqs. (1.1) and (1.2) it follows that the frequency f' registered by the receiver depends on the source speed in a non-linear way. Employing Taylor series we get

$$\frac{c}{c \mp v_\mathrm{S}} = \frac{1}{1 \mp v_\mathrm{S}/c} = 1 \pm \frac{v_\mathrm{S}}{c} + \left(\frac{v_\mathrm{S}}{c}\right)^2 \pm \left(\frac{v_\mathrm{S}}{c}\right)^3 + \ldots,$$

so if $v_\mathrm{S} \ll c$, we can take into account only the first two terms in the series and Eq. (1.1) and (1.2) reduce into

$$f' = \frac{c \pm v_\mathrm{S}}{c} f, \tag{1.3}$$

where the plus sign corresponds to the approaching source and the minus sign corresponds to the receding source.

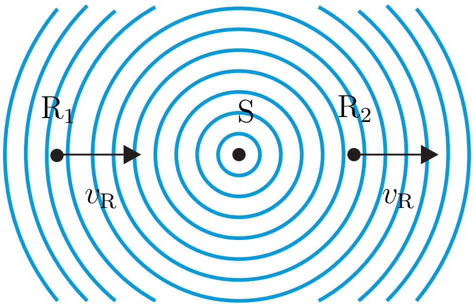

Figure 1.2: Moving receiver.

Situation where the source is at rest and the receiver moves is shown in Fig. 1.2. As the receiver R_1 approaches the source with speed v_R, the time between its encountering two following crests decreases. The relative wave speed with respect to the receiver $c' = c + v_\mathrm{R}$, whereas the wavelength does not change, and it reads $\lambda = cT$. For the frequency registered by the receiver it then holds

$$f' = \frac{c'}{\lambda} = \frac{c + v_\mathrm{R}}{cT} = \frac{c + v_\mathrm{R}}{c} f. \tag{1.4}$$

Relative wave speed with respect to the receiver R_2, which recedes from the source, is $c' = c - v_\mathrm{R}$, so for the registered frequency it holds

$$f' = \frac{c - v_\mathrm{R}}{c} f. \tag{1.5}$$

It can be seen from Eqs. (1.3), (1.4), and (1.5) that if the source/receiver speed is much smaller than the speed of sound, the Doppler frequency shift for both the moving source or receiver can be described using one formula

$$f' = \left(1 \pm \frac{v}{c}\right) f = f \pm \frac{f}{c} v, \tag{1.6}$$

where v stands for source/receiver speed, plus sign for approaching and minus sign for recession.

If both the source as well as the receiver move at the same time, the frequency registered by the receiver $f' = c'/\lambda'$. Combining Eqs. (1.1), (1.2), (1.4), and (1.5) results in

$$f' = \frac{c \pm v_\mathrm{R}}{c \mp v_\mathrm{S}} f. \tag{1.7}$$

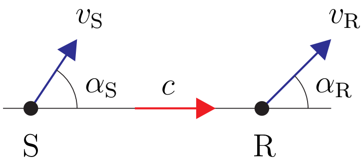

Figure 1.3: Oblique motion.

For example, it follows from here that if source and receiver move with the same speed $v = v_\mathrm{S} = v_\mathrm{R}$ the same direction (they are at rest with respect to each other), the receiver registers frequency

$$f' = \frac{c + v}{c + v} f = f.$$

All previous relations have been derived assuming that the receiver and the source move along the same line. If the receiver moves in an oblique direction with respect to the sound wave propagation, see Fig. 1.3, only the component of its velocity in the direction of the wave propagation $v_\mathrm{R} \cos \alpha_\mathrm{R}$ applies, where α_R is the angle between the direction of the receiver velocity and the wave propagation. Similarly, for the source moving in an oblique direction, only the velocity component $v_{\mathrm{S}} \cos \alpha_{\mathrm{S}}$ applies. The formula describing the Doppler effect in this general case has the form

$$f' = \frac{c - v_{\mathrm{R}} \cos \alpha_{\mathrm{R}}}{c - v_{\mathrm{S}} \cos \alpha_{\mathrm{S}}} f. \tag{1.8}$$

The wave propagation direction and the angles α_{R} and α_{S} are related to the line SR; for the receiver at the time of the wave registration, and for the source at the time of the wave generation. This must be taken into the account in the case where the angles α_{R} and α_{S} are time-dependent. The relation (1.8) shows that if the receiver or source moves perpendicularly with respect to their connecting line, the corresponding motion does not affect the registered frequency.

It can be seen from the relations (1.1), (1.2), and (1.4), (1.5) that the frequency registered by the receiver differs for moving source and moving receiver. It also follows from the relation (1.7) that if the source as well as the receiver move, the registered frequency is not an explicit function of the relative velocity $v_{\mathrm{R}} \pm v_{\mathrm{S}}$. It means that principle of relativity does not apply for acoustic Doppler effect –all the reference frames are not equivalent for the acoustic wave propagation description.

There is a distinguished reference frame—the one in which the fluid, in which the wave propagates, is at rest.

1.3 Experiment

Within the experiment, a toy-train is employed which carries the ultrasonic (USC) transmitter–source, or receiver, towards (from) a fixed receiver (transmitter). The measurement is performed using a controller, see Fig. 1.4, which measures both the speed of the train and the frequency of the USC signal perceived by the receiver. The train speed is derived from the time the light barrier is shadowed by a train-carried shade, its length is firmly programmed in the controller, and the instrument directly displays the speed in meters per second (the experimenter is left to check that the shade is oriented straight along the train axis). The speed measurement starts automatically when the light barrier is shadowed, the device measures the shadowing time with an accuracy of microseconds. The measurement of frequency is started by pushing the button “Measure” on the touch-screen of the controller. The controller counts the number of cycles of the USC signal during the period of 1 second. The controller displays the measured frequency with an accuracy of ca. ±2 Hz.

Cables and individual devices are not needed to be disconnected after the measurement, so do not do so unnecessarily. If some cables happen to be disconnected (or something does not work), the connection procedure is given below.

Plug the USC receiver to the input BNC connector of the USC unit (14, Fig. 1.5). Connect the USC unit output (13, Fig. 1.5) to BNC input of the controller, see Fig. 1.4, using a shielded cable. Plug the USC transmitter into the terminal TR1 of the USC unit (10, Fig. 1.5) and using the button 4, switch the USC unit into the continuous regime (indicated with LED **Cont.**). Check that the cable from light barrier is connected into the input 4 of the controller, see Fig. 1.4.

1.4 Procedure

1. Check the connection of the individual instruments, get familiar with the control of the toy-train.

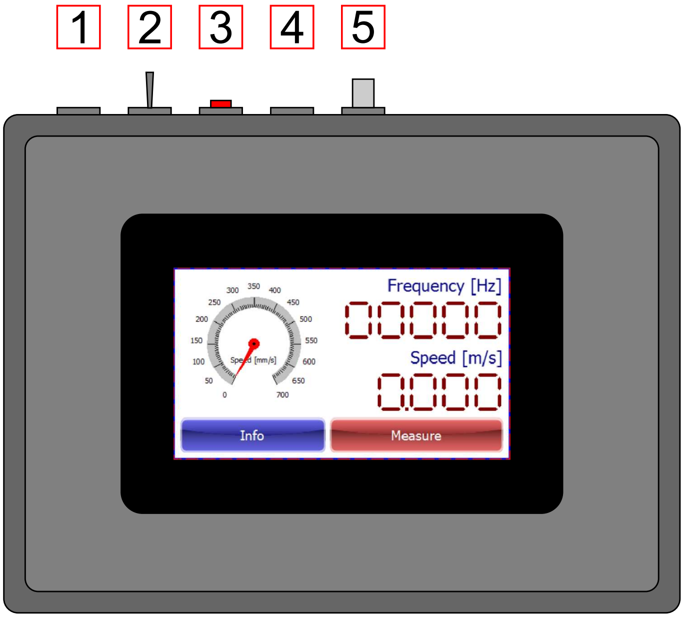

Figure 1.4: Controller. 1 – Power supply connector 5 V, 2 – Switch on/off, 3 – RESET button, 4 – Three-pin terminal for the connection of light barrier, 5 – BNC connector for connecting the USC unit.

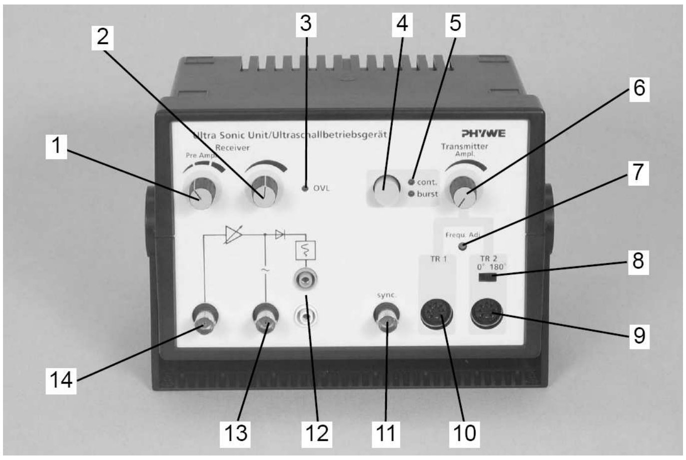

Figure 1.5: Ultrasonic (USC) unit. 1 - Rotary switch for input signal amplification selection; 2 - Potentiometer for the input signal amplification adjustment; 3 - Overload LED indicator (OVERLOADED); 4, 5 - Operation mode selection button with LED indicator: cont. indicates the continuous mode, burst indicates the burst mode; 6 - Potentiometer for the output signal amplitude adjustment; 7 - USC frequency adjustment; 8 - Switch for the phase reversal of the output USC signal; 9, 10 - Terminals for the connection of USC transmitters; 11 - Analog power output; 12 - Output of amplified and rectified signal of the USC receiver; 13 - Output of amplified signal of the USC receiver; 14 - Input for the connection of USC receiver.

2. Adjust the gain of the USC unit (controls 1 and 2, see Fig. 1.5) and the output signal amplitude (potentiometer 6) in such a way that the controller measures the frequency even at end-positions of the toy-train. It may happen that if the toy-train is close to the stationary receiver/transmitter, the USC unit input is overloaded (indicated by LED **OVL**). This situation does not influence the measurement. Place the light barrier to a position where the toy-train speed is more-or-less constant.

3. For the toy-train at rest, several speeds and both the directions, measure the USC frequency for moving USC transmitter or receiver. From the repeated measurements for each speed/direction, calculate the average speed and frequency.
4. If you have left time, you can repeat the measurement with moving receiver/transmitter.
5. The speed of sound is temperature-dependent, so read the temperature on the wall thermometer display and calculate the speed of sound using the formula $c = 331.06 + 0.61\,t$ [m/s, °C].
6. Compare quantitatively the theoretical and measured data. You can use the following procedure.

   In one graph, plot the dependence of the measured frequency f' on the toy-train speed, where for recession, consider the speed negative. Employing the least squares method, e.g., implemented as *An universal tool for plotting graphs - least squares method* at server

   https://planck.fel.cvut.cz/praktikum/

   approximate the measured data with a straight line (polynomial of the first degree). Compare its tangent with the calculated ratio f/c, see the relation (1.6). The same procedure can be used for both the moving transmitter or receiver.

1.5 References

1. Jiří Bajer: Mechanika 3, *Univerzita Palackého v Olomouci*, Olomouc, 2006.
2. Jan Horský a kolektiv: Mechanika ve fyzice, *Academia*, Praha, 2001.

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