Laboratory experiment

Measurement of focal lengths of lenses

1.1 Task

1. Measure the focal lengths of at least two converging lenses and compare the measured results with the values indicated on the lenses.
2. Measure the focal lengths of at least two diverging lenses and compare the measured results with the values indicated on the lenses.

1.2 Theory

1.2.1 Image formation by ideal lenses

After the plane mirror, the lens is probably the best known and most used optical device. In the simplest case, it consists of two spherical refractive surfaces whose central axes coincide to form the so-called optic axis. If the refractive surfaces are so close together that their distance (the thickness of the lens) can be neglected in relation to other dimensions (e.g., the radii of the refractive surfaces), we speak of a so-called thin lens, to the description of which we shall confine ourselves here.

Converging lens

The basic property of the so-called ideal converging lens is, see Fig. 1.1, that when a beam of light parallel to the optic axis is incident on it, the light passing through the lens is focused to a single point F' lying on the optic axis at a distance f from the lens. The point F' is called the focal point, the distance f the focal length. Similarly, light emitted from a point source lying at point F at distance f on the optic axis in front of the lens forms a beam parallel to the optic axis behind the lens.

The image I of a point O (object) lying at the (object) distance a in front of an ideal converging lens at a distance y from the optic axis can be found geometrically by ray construction, see Fig. 1.2, as the intersection of at least two of the three principal rays passing through the point O. Ray 1 comes parallel to the optic axis and refracts (see above) so that it passes through the focal point F'. Ray 2 arriving at the center of the lens is not refracted (since the refractive surfaces are parallel here). Ray 3 passing through the focal point F has a direction parallel to the optic axis behind the lens (see above).

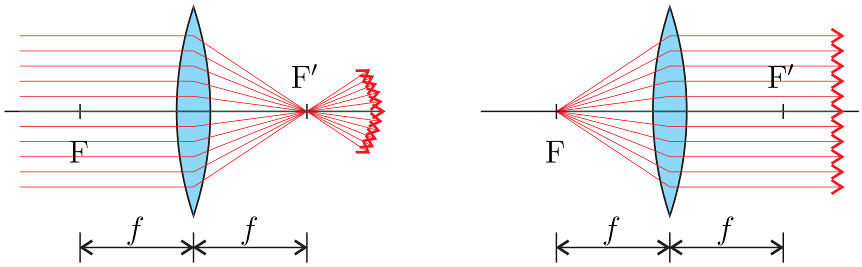

Figure 1.1: Converging lens.

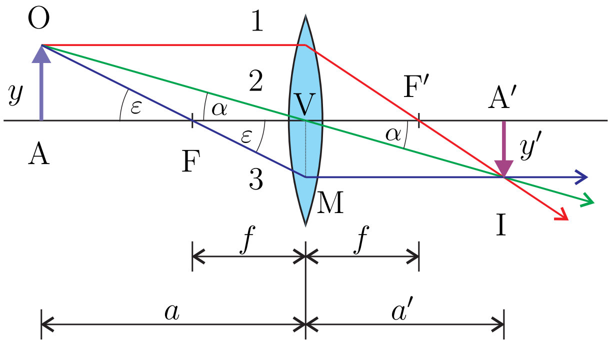

Figure 1.2: Image formation by a converging lens.

We can easily determine the position of the image I of the object O by calculation, but for this we first need to introduce the *sign rules*[1].

- If the object is on the side of the lens where the light is incoming, the object distance a is positive, otherwise it is negative.
- If the image is on the side of the lens where the light is outgoing, the image distance a' is positive, otherwise it is negative.
- If the object or image is "above" the optic axis, we consider the lateral distances y, y' positive, if the object or image is "below" the optic axis, we consider the lateral distances y, y' negative.
- The focal length f of a **converging lens** is positive.

From the similar triangles OAF and MVF (see Fig. 1.2) it follows

$$\tan\varepsilon = \frac{y}{a-f} = \frac{-y'}{f}, \tag{1.1}$$

similarly, from the triangles OAV and IA′V it follows

$$\tan\alpha = \frac{y}{a} = \frac{-y'}{a'}. \tag{1.2}$$

Combination of Eqs. (1.1) and (1.2) results in the object-image relationship

$$\frac{1}{a} + \frac{1}{a'} = \frac{1}{f}. \tag{1.3}$$

[1] Several different sets of sign rules can be found in textbooks, and the formulas of geometric optics have different forms for each set of sign rules.

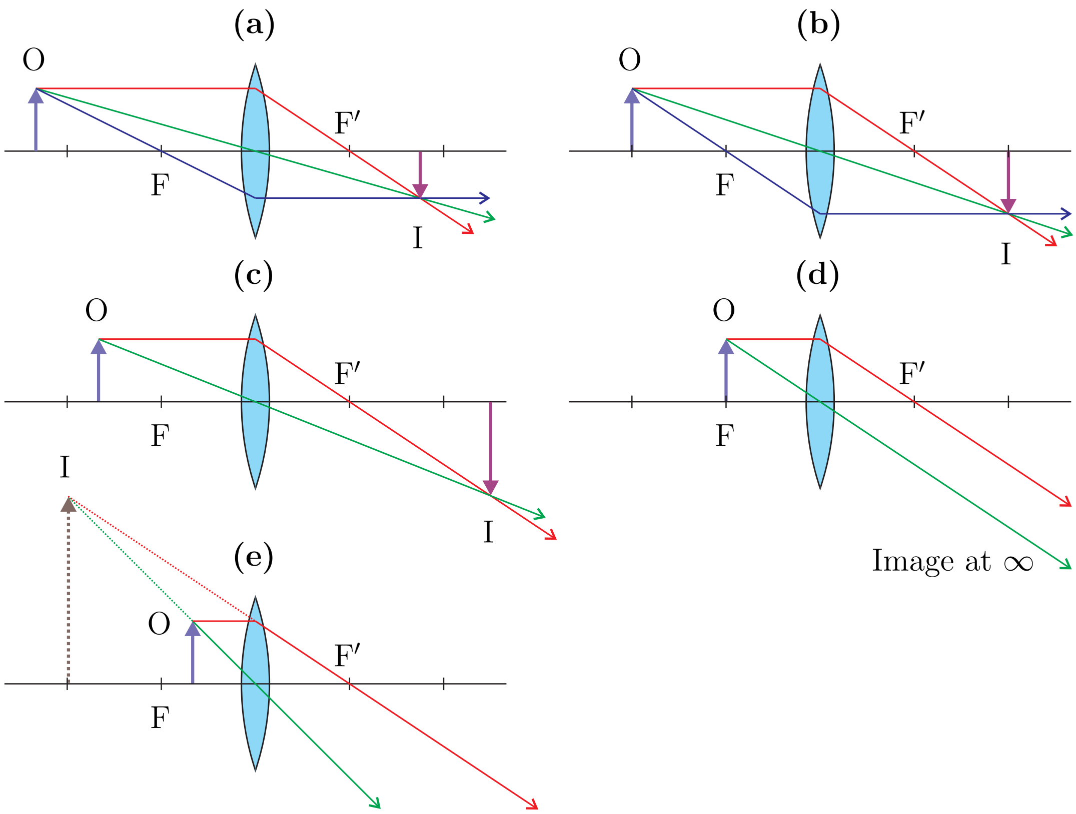

Figure 1.3: Examples of image formation by a converging lens.

For the so-called lateral magnification it holds

$$m \equiv \frac{y'}{y} = -\frac{a'}{a}, \tag{1.4}$$

see Eq. (1.2). If $m > 0$, we call the image *erect*, if $m < 0$, the image is called *inverted*.

It can be easily shown that for the focal length of a thin lens so-called lensmaker's equation applies, which reads

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right), \tag{1.5}$$

where n is the relative index of refraction of the lens material related to the surrounding material (e.g., air[2]), r_1 is the radius of curvature of the refractive surface closer to the object, and r_2 is the radius of curvature of the surface farther away. The radius of curvature can be positive or negative, according to the sign rules used here:

- if the center of curvature of the refractive surface is on the side of the outgoing light, the radius is positive, otherwise it is negative.

Examples of the graphic construction of image formation by a converging lens are shown in Fig. 1.3. From the figure and the relations[3]

$$m = \frac{1}{1 - a/f}, \qquad a' = \frac{af}{a-f} \tag{1.6}$$

[2]Since the index of refraction of air is close to one, in this case n is the absolute index of refraction of the lens material.

[3]Derived from Eqs. (1.1) and (1.2).

it is apparent that: a) if $a > 2f$, the image is inverted and demagnified, b) if $a = 2f$, the image is inverted and it has the same size as the object, c) if $f < a < 2f$, the image is inverted and magnified. In cases a) – c) we speak about the so-called real image, which is formed by the intersection of real rays and can be captured on a screen. In case d), where $a = f$, the image is formed in infinity (the rays do not intersect at a finite distance). In case e), where $0 < a < f$, the rays coming from the object O diverge behind the lens. It follows from Eqs. (1.6) that the image is erect and magnified and the image distance a' is negative – according to the sign rules the image is at the same side as the object. The figure shows that the image is not formed by the intersection of real rays (which are diverged by the lens), but by their backward extensions. In this case we speak about the so-called virtual image, which is not formed by the intersection of real rays; if we were to place a screen in that spot, no image would be observable on it. In order to observe the virtual image, we need to use another optical system (e.g., an eye or a camera) that is able to focus the diverging rays into a real image (e.g., formed on the retina of the eye or on a CCD chip). The backward extension of the real refracted rays is used to construct the virtual image, see Fig. 1.3 e).

Diverging lens

Complementary to the converging lens is the diverging lens. The basic property of the so-called ideal diverging lens is, see Fig. 1.4, that when a beam of light parallel to the optic axis is incident on it, the light is refracted by the lens as if it were emitted from a single point (focal point) F′ lying on the optic axis at a distance f from the lens. Similarly, rays directed to a focal point F lying at a distance f on the optic axis behind the lens are refracted so that they form a beam parallel to the optic axis behind the lens.

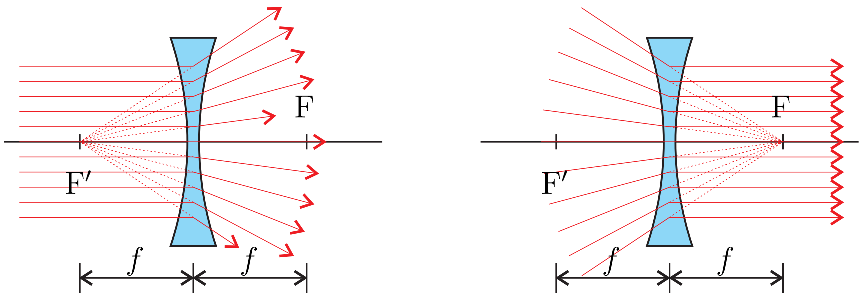

Figure 1.4: Diverging lens.

In the case of an ideal thin diverging lens, the image I of object O can be found graphically by ray construction in a similar way as in the case of a converging lens. Since the rays emitted from object O always diverge behind the diverging lens, we find the image as the intersection of the backward extensions of the principal rays, see Fig. 1.5. It is clear from the figure that the image in the case of a diverging lens always lies on the same side as the object, is virtual and has the same orientation as the object.

The position and lateral magnification of the image can also be found in the case of a diverging lens using the relations (1.6) based on the object-image equation (1.3) using the sign rules introduced above, the only difference being that

- focal length f of the **diverging lens** is negative.

The focal length of a thin diverging lens can again be calculated using the lensmaker's equation (1.5).

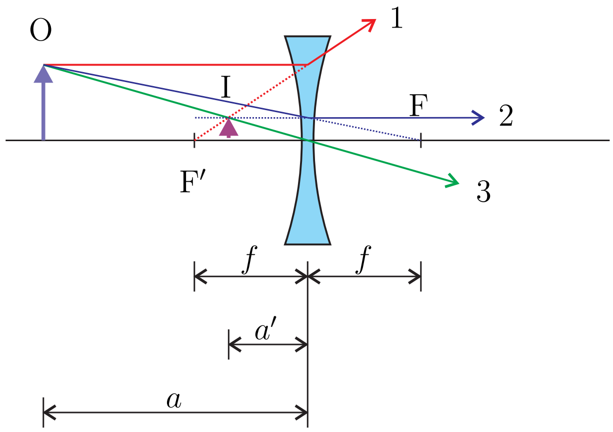

Figure 1.5: Image formation by a diverging lens.

Combination of multiple lenses

If an object O is placed in front of a set of two (more) lenses with coincident optic axes, it is possible to determine the position and size of the final image I (formed by the lens further away from the object) by successive numerical or graphical solutions. The lens closer to the object is denoted by index 1, the lens farther away by index 2. The direction "in front of the lens" is the direction to O.

1. We ignore lens 2. Let us denote the distance of the object $\mathrm{O}\equiv\mathrm{O}_1$ in front of lens 1 as a_1 and from the relations (1.6) we calculate for the image I_1 its position a_1' and the lateral distance from the optic axis y_1'.

2. The image I_1 is the object O_2 for the second lens $\mathrm{I}_1 \equiv \mathrm{O}_2$. We ignore lens 1. We denote the distance of object O_2 from lens 2 by a_2. By sign rules, if O_2 is in front of lens 2, then $a_2 > 0$, otherwise $a_2 < 0$, and O_2 is a virtual object. From the relations (1.6) we calculate for the image $\mathrm{I}_2 \equiv\mathrm{I}$ its position a_2' and the lateral distance from the optic axis y_2'.

Example of geometric construction of a image formation by a combination of converging and diverging lens is shown in Fig. 1.6.

1.2.2 Lens aberrations

The imaging system should ideally display a point object as a point, a straight line as a straight line (and therefore a plane as a plane), and the image properties should not depend on the wavelength (color of the light). Such an imaging system is called ideal and does not exist in practice. Any projection by a real imaging system is burdened with defects–aberrations; in practice, these aberrations must be corrected so that their magnitude in the image is less than the resolution of the image receiver (e.g., sensor).

Due to the so-called spherical aberration, the converging lens does not project a point lying on the optic axis as a point, but as a ring. This is due to the fact that rays incident from this point on the lens at different angles intersect the optic axis at different distances behind the lens (rays close to the optical axis intersect further away). The spherical aberration therefore manifests itself in wide beam imaging. When imaging a point lying outside the optic axis with a wide beam, a similar aberration, the so-called coma, appears, where the image of the point resembles the tail of a comet.

The astigmatism manifests itself in the imaging of points lying outside the optic axis even with a narrow beam, so that the rays lying in two mutually perpendicular planes are focused at different points.

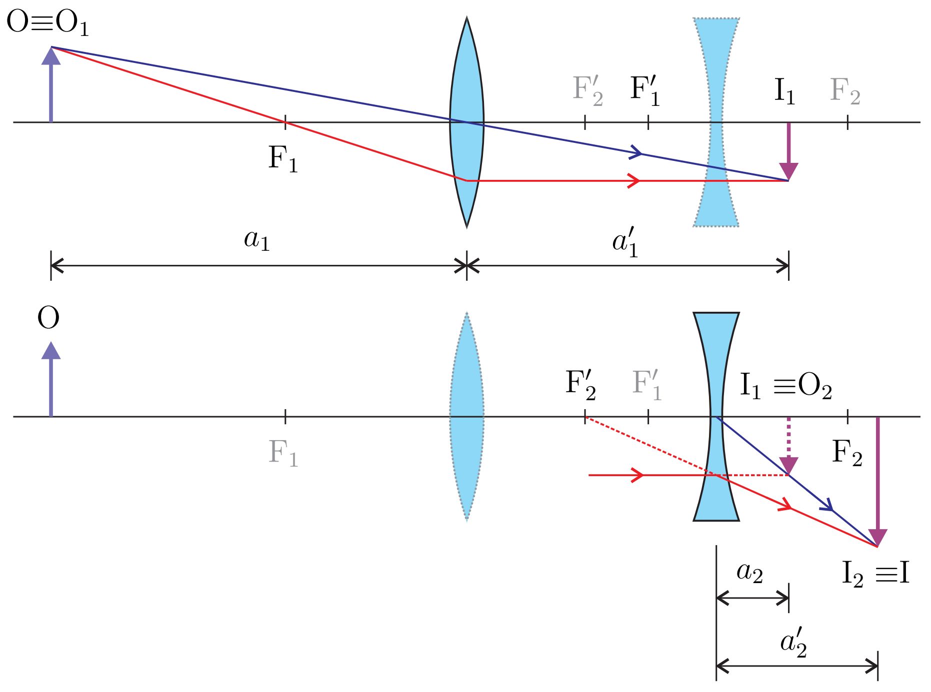

Figure 1.6: Example of image formation by a combination of two lenses. As the image I_1 is behind the lens 2, it represents its virtual object O_2 (the object distance a_2 is negative).

Curvature of the field of the image is an optical aberration where the image of a plane perpendicular to the optic axis does not lie in a plane, but on a kind of curved surface. It usually manifests itself in such a way that if the centre of the image is sharp, its edges are blurred and vice versa. Furthermore, the image may be distorted by the fact that points at different distances from the optic axis are displayed with different lateral magnifications.

The chromatic aberration is due to the fact that the index of refraction of the lens material depends on the wavelength of light and therefore the focal length, see the lensmaker's equation (1.5), and therefore the properties of the image depend on the wavelength.

1.3 Experiment

The measurement of the focal lengths of lenses is carried out on an optical bench equipped with a light source illuminating a slide (which serves as the object), converging and diverging lenses (which can be considered thin) with different focal lengths and a screen. The position of the object, the lenses and the screen can be varied on the optical bench and read on a scale with millimeter division.

The light source is switched on by pressing the blue button on the light source power module, switching off is done by pressing the red button.

A color filter can be placed between the object and the light source, eliminating the chromatic aberration that is especially noticeable at higher magnifications.

1.3.1 Measurement of the focal length of a converging lens

The relative position of the slide (object), the converging lens and the screen is adjusted so that the sharpest possible image of the object[4] can be observed on the screen, see Fig. 1.7. The focal

[4] For a real image to be observable on the screen, $a > f$ must hold. The image will be inverted and magnified or demagnified, see Fig. 1.3a-c.

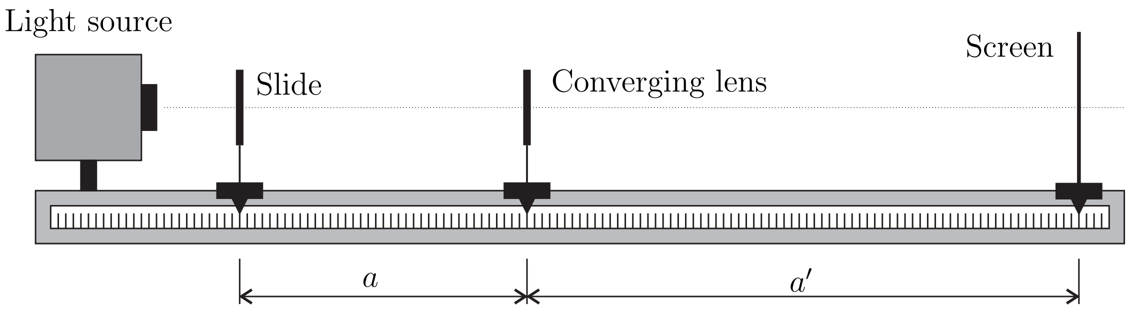

Figure 1.7: Measurement of the focal length of a converging lens.

length of the converging lens is then calculated from the object-image equation (1.3) as

$$f = \frac{aa'}{a + a'}. \tag{1.7}$$

According to the sign rules, in this case, the distances a, a' are positive. The focal length can also be calculated from the lateral magnification from Eq. (1.4) as

$$f = a\frac{|m|}{1 + |m|} = a'\frac{1}{1 + |m|}. \tag{1.8}$$

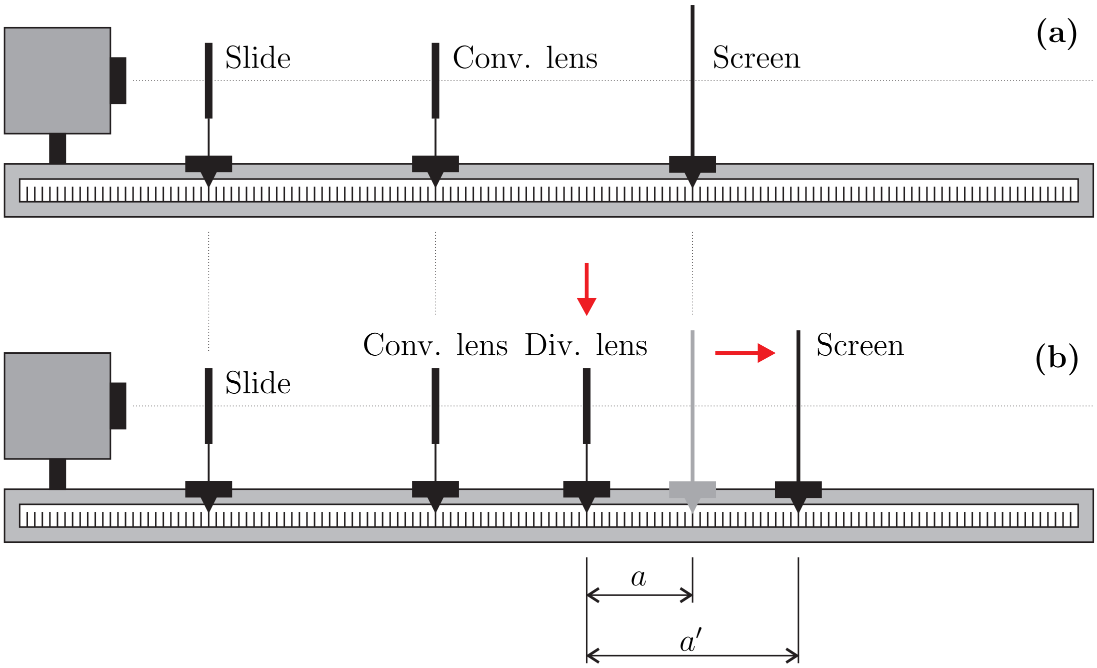

Figure 1.8: Measurement of the focal length of a diverging lens.

1.3.2 Measurement of the focal length of a diverging lens

Measurement of the focal length of a diverging lens is slightly more complicated than that of a converging lens, since the diverging lens itself does not produce a real image that can be captured on a screen. However, we can use combination of two lenses, as it is shown in Fig. 1.6. The actual procedure is then shown in Fig. 1.8.

In the first step, a sharp image of the object is projected on a screen using the auxiliary converging lens, as described in Sec. 1.3.1 (see Figure 1.8a). Note the position of the screen on the optical bench. Then, between the converging lens and the screen, insert the investigated diverging lens[5] and find the new position of the screen for which the image is sharp again, see Fig. 1.8b. The virtual object is located at the original position of the screen and it is projected by the diverging lens as real at the new position of the screen.

As the object is virtual, it applies $a < 0$, the image is real, thus $a' > 0$, and for the diverging lens it applies $f < 0$. From the object-image equation (1.3) we obtain

$$f = \frac{a'|a|}{|a| - a'}. \tag{1.9}$$

The focal length can also be calculated from the lateral magnification, using the relation (1.4) we can write

$$f = |a|\frac{m}{1-m} = a'\frac{1}{1-m}. \tag{1.10}$$

1.4 Procedure

1.4.1 Measurement of the focal length of converging lenses

1. Switch on the light source by pressing the blue button. Select an object (slide) and place it in the appropriate holder on the optical bench. **Please, try to touch only the frame of the slide if possible and, in general, do not touch any translucent parts of the optical elements**. Place the object (and all optical elements in general) on the optical bench so that their axes are as coincident as possible – this will eliminate unnecessary deterioration of the quality of the resulting image. A color filter can be placed between the object and the light source to eliminate chromatic aberrations that are especially noticeable at higher magnifications. Place the screen on the optical bench so that its surface is perpendicular to the axis of the bench.

2. Choose a converging lens (focal lengths on converging lenses are indicated with the sign "+") and place it on the optical bench. Before the actual measurement, test that the projection has the properties shown in Fig. 1.3.

3. Measure the focal length of the lens according to paragraph 1.3.1 for at least 10 different object (image) distances. You can do this by placing the screen in different positions on the optical bench in succession, searching for the position of the lens such that the image on the screen is sharp (this will prevent you from the incidental setting the lens position such that the image is virtual). Obviously, your measurement will be as accurate as you can decide when the image on the screen is "sharp." This decision will be made more difficult by the presence of imaging aberrations.

4. Repeat the measurement (from point 2) for another converging lens.

[5]From the figure 1.6 it is clear (and can be verified by substituting it into the object-image equation) that for the image to be real, the virtual object (at the original position of the screen) must lie between the diverging lens and its focal point (its position is not yet "known").

[5]We can do something different – move the screen to some new position (away from the light source) and find the position of the diverging lens such that the image is as sharp as possible in the new position of the screen.

5. Calculate the focal lengths of the lenses (and their uncertainties) from the measured values using the formula (1.7) and compare the measured values with the values indicated on the lenses.

1.4.2 Measurement of the focal length of diverging lenses

1. The procedure is described in Sec. 1.3.2. Choose a converging lens and place it on the optical bench together with the screen, see Fig. 1.8a so that the image on the screen is sharp. Note the position of the screen.

2. Choose a diverging lens (the focal length is indicated on the diverging lens with the sign "–") and place it between the converging lens and the screen and find the new position of the screen, see Fig. 1.3.2b, such that the image on the screen is sharp. Note the position of the diverging lens and the new position of the screen. Repeat this step at least $10\times$ for different positions of the diverging lens. Alternatively, this step can be performed by first setting the new position of the screen (away from the converging lens) and then placing the diverging lens between it and the converging lens in such a position that the image is sharp. This avoids the possible situation where the image is virtual.

3. Repeat the measurement (step 2) for another (the second one) diverging lens.

4. Use the formula (1.9) to calculate the focal lengths of the lenses (and their uncertainties) and compare the measured values with the values indicated on the lenses.

1.5 References

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