Laboratory experiment

Measurement of permittivity of dielectric materials

1.1 Tasks

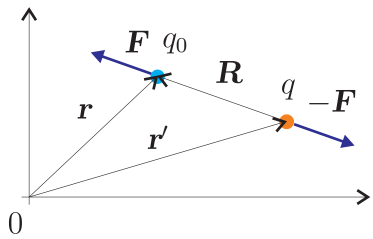
- Determine the relative permittivity of one or two samples of dielectric materials.
- For each sample (in each case in one graph) plot the dependence of the charge of the capacitor on the voltage with and without the dielectric sample placed between the plates of the capacitor.

1.2 Introduction

1.2.1 Coulomb's law

Let there be a (source) point particle with electric charge q in the vacuum and with the position vector $\boldsymbol{r}'$ and another (test) point particle with charge q_0. According to Coulomb's law, an electrostatic force acts on the test particle which reads

$$\boldsymbol{F} = \frac{q_0 q}{4\pi\varepsilon_0}\frac{\boldsymbol{R}}{|\boldsymbol{R}|^3}, \tag{1.1}$$



where $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$ is the position vector of the test charge with respect to the source charge. In the SI system, the unit of charge is coulomb (C), the quantity ε_0 is called the electric constant (permittivity of free space) with the value

$$\varepsilon_0 == 8.854\,187\,8128(13) \times 10^{-12}\,\mathrm{C^2N^{-1}m^{-2}},$$

If both the charges have the same sign, the vectors $\boldsymbol{F}$ and $\boldsymbol{R}$ have the same direction (charges repel each other), in the case where the charges have opposite signs, the vectors $\boldsymbol{F}$ and $\boldsymbol{R}$ point in the opposite directions and the charges attract each other. According to the Newton's third law, the force $-\boldsymbol{F}$ acts on particle with charge q,

If we change the position of the test charge in space (by changing the position vector $\boldsymbol{r}$), the magnitude and direction of the force acting on it will also change. The magnitude and direction of this force are also related with the value and sign of the test charge q_0. In order to be able to simply describe the force action on any test charge, we introduce the so-called electric field by the defining relation

$$\boldsymbol{E} \equiv \frac{\boldsymbol{F}}{q_0},$$

which is thus numerically equal to the force acting on the unit charge[1]. The vector $\boldsymbol{E}$ has the same direction as the force $\boldsymbol{F}$ acting on the positive charge q_0. We then say that the charge q creates an electrostatic field around it with intensity $\boldsymbol{E} = \boldsymbol{E}(\boldsymbol{r})$. In the case of a single source charge, the substituting into the formula (1.1) yields in

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{q}{4\pi\varepsilon_0}\frac{\boldsymbol{r}-\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^3}.$$

If there are more source point charges distributed in space, according to the superposition principle, the electric field they create in their surroundings can be calculated as

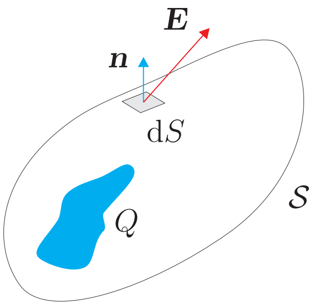
$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i\cdot(\boldsymbol{r}-\boldsymbol{r}'_i)}{|\boldsymbol{r}-\boldsymbol{r}'_i|^3}, \tag{1.2}$$

where $\boldsymbol{r}'_i$ are the position vectors of the individual charges q_i.

In many practical cases[2] the charges are distributed in space so densely that their contributions cannot be added together by a summation. In these cases, we introduce the so-called charge density (length, area, volume) and "add their contribution" using the integral, as if the charge were a continuous quantity. Let's give an example. Let the charges be on some surface $\mathcal{S}$. We divide this area into individual elementary ones $\mathrm{d}S'$, where each of them will possess an elementary charge $\mathrm{d}q = \sigma\mathrm{d}S'$, where $\sigma = \sigma(\boldsymbol{r}')$ is the surface charge density (charge per unit area). We then get for the electric field at $\boldsymbol{r}$ the result

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\iint\limits_{\mathcal{S}}\frac{\sigma(\boldsymbol{r}')\cdot(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\mathrm{d}S'. \tag{1.3}$$

1.2.2 Gauss's law

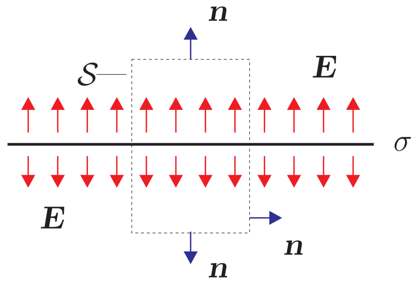


Calculating integrals of type (1.3) can be quite difficult in many cases. However, in some cases, Gauss's law can be used to calculate the electric field. It has the form

$$\oiint\limits_{\mathcal{S}}\boldsymbol{E}\cdot\mathrm{d}\boldsymbol{S} = \frac{Q}{\varepsilon_0}. \tag{1.4}$$

The formula (1.4) states that the flux of the electric field vector through any closed (Gaussian) surface $\mathcal{S}$ is equal to the ratio of the total charge Q within this surface and the electric constant. The elementary vector $\mathrm{d}\boldsymbol{S} = \boldsymbol{n}\,\mathrm{d}S$ is oriented by the unit vector $\boldsymbol{n}$ perpendicular to the area $\mathrm{d}S$ outwards from the closed surface $\mathcal{S}$.

The Gauss's law is equivalent for the electrostatic problems to the Coulomb's law. It can be conveniently used to calculate the electric field in cases where the charge distribution has a symmetry (planar, cylindrical or spherical). Here are two simple examples.



Consider an infinite, uniformly charged planar surface with surface charge density $\sigma > 0$. A distinctive direction to the planar surface is the normal to it, so the electric field vector must point perpendicular to the planar surface. Since it is positively charged,

[1] It follows from the definition that the unit of electric field in SI system is newton per coulomb ($\mathrm{N\,C^{-1}}$), but more often volt per meter is used ($\mathrm{V\,m^{-1}}$), which has the same size ($1\,\mathrm{N\,C^{-1}} = 1\,\mathrm{V\,m^{-1}}$).

[2] More precisely, for the vast majority of cases...

it will certainly point away from it. For the Gaussian integration surface we choose the surface of the cylinder, which the plane intersects and whose bases are parallel with the charged plane. If the base area of this cylinder is ΔS, $Q = \sigma\Delta S$ applies to the total charge enclosed by the integration surface. Thus, we can write for the surface integral in the formula (1.4) that

$$\oiint_{\mathcal{S}} \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{S} = \iint_{\text{bases}} \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{S} + \iint_{\text{side}} \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{S}. \tag{1.5}$$

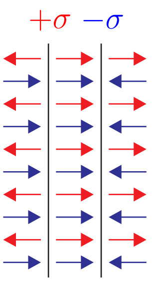
Since $\boldsymbol{E} \perp (\boldsymbol{n}\,\mathrm{d}S)$ holds everywhere on the cylinder side, the second of the integrals (1.5) is zero. On the bases, the electric field vector has the direction of the outer normal and (due to symmetry) its magnitude is constant, so

$$\oiint_{\mathcal{S}} \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{S} = \iint_{\text{bases}} \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{S} = \iint_{\text{bases}} E\mathrm{d}S = E\iint_{\text{bases}} \mathrm{d}S = 2E\Delta S.$$

Substitution to the Gauss's law then results in

$$2E\Delta S = \frac{\sigma\Delta S}{\varepsilon_0} \qquad \Rightarrow \qquad E = \frac{\sigma}{2\varepsilon_0}. \tag{1.6}$$

The magnitude of the electric field vector is therefore constant, it does not decrease with the distance from the charged plane. This result may seem paradoxical at first glance, it is caused by the fact that we consider an infinite plane (and thus an infinite charge). In a more realistic case, if we considered a limited planar surface, the result (1.6) would apply approximately only in its close vicinity.

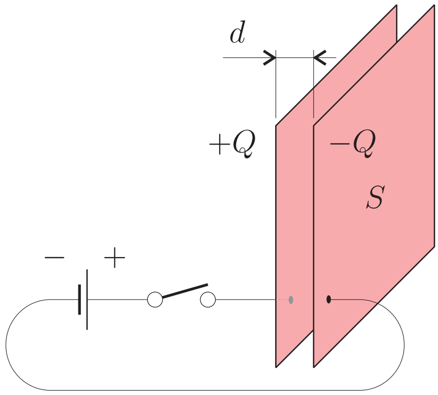


Let's now consider the case of two parallel planes which are uniformly charged by surface charge densities of the same size but of opposite sign. Each of the planes in its vicinity creates the electric field, the magnitude of which is given by the relation (1.6). In the case of the positively charged plane the electric field vector points outside from the plane, in the case of the negatively charged plane it points towards it. This means (see the figure) that between the planes, the contributions are added to each other and they are subtracted from each other outside. Thus, between the planes, the magnitude of the electric field reads

$$E = \frac{\sigma}{\varepsilon_0}, \tag{1.7}$$

and its magnitude is zero outside.

1.2.3 Capacitor, capacitance



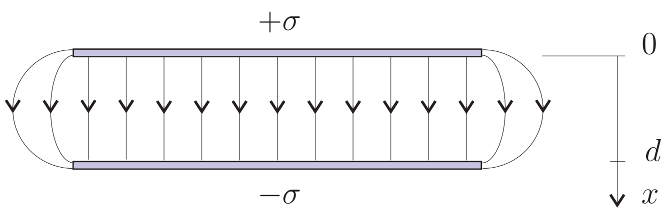
A capacitor is an electronic component in which energy can be stored in the form of an electric field. In practice, it most often consists of two conductors (electrodes), which are placed close to each other, but they are insulated from each other. These can be represented by two plane-parallel plates with an area of S placed at a distance d from each other, see the figure, then we are talking about the so-called plate capacitor.

If we connect a battery to such a system, see the figure, free charge carriers (negatively charged electrons) are attracted to the positive terminal of the battery and they are thus pumped out of the left capacitor plate, where positively charged ions of the crystal lattice remain. The left plate thus starts to be charged with a more positive charge, than the right one. This creates an electrostatic field in the capacitor, which attracts electrons from the negative terminal of the battery to the right plate. The whole process continues until the capacitor is charged to the battery voltage U. On the left plate there is an accumulated positive charge Q, on the right plate there is a negative charge $-Q$. A characteristic feature of the capacitor is the capacitance, which describes the relationship between the voltage and charge accumulated on the capacitor[3]. We will investigate it for the case of the above-mentioned plate capacitor.

If the distance between the charged plates with respect to their dimensions is very small, it can be assumed that the charge is distributed on them more or less evenly, which creates a more or less homogeneous electrostatic field between the plates, for the magnitude of its intensity will roughly apply the relation (1.7).

The voltage (between points A and B) is defined as the work done by the electrostatic field when moving the unit charge from the point A to the point B, namely,

$$U = \int_{\mathbf{r}_A,\mathcal{L}}^{\mathbf{r}_B} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{r}. \tag{1.8}$$



Since the electrostatic field is a conservative field, the work done does not depend on the specific integration path $\mathcal{L}$. The integral (1.8) is calculated along a line of force from the positively charged electrode to the negatively charged electrode, so we get

$$U = \int_0^d E\,\mathrm{d}x = \int_0^d \frac{\sigma}{\varepsilon_0}\,\mathrm{d}x = \frac{\sigma d}{\varepsilon_0} = \frac{d}{\varepsilon_0 S} Q, \tag{1.9}$$

where we used the relation for the surface charge density $\sigma = Q/S$. It can be seen from the formula (1.9) that there is a direct relationship between the voltage between the electrodes and the accumulated charge. The proportionality constant is called the capacitance of the capacitor, we introduce it by a definition relation

$$C = \frac{Q}{U} \tag{1.10}$$

and numerically it is thus the charge accumulated on the capacitor at the potential difference 1 V between the electrodes. The unit of capacity is farad = coulomb per volt (F). Therefore, in the case of a plate capacitor it holds

$$C_{\mathrm{vac}} = \frac{\varepsilon_0 S}{d}, \tag{1.11}$$

where the index "vac" indicates that we consider a materialless environment (vacuum) between the electrodes. The capacity of the plate capacitor is thus greater the larger the area of the electrodes and the closer they are to each other. The formula (1.11) holds only approximately, in its derivation the assumption was used that the charge on the plates is distributed evenly and that the electric field between them is homogeneous. In fact, the so-called scattering phenomenon occurs at the edges of the plates, the formula (1.11) holds with an accuracy of about 20 % if $d/\sqrt{S} \sim 0.1$ and with an accuracy of about 2 % if $d/\sqrt{S} \sim 0.01$.

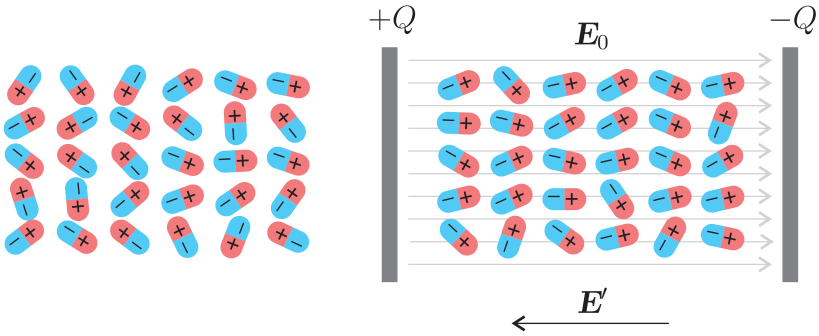
In addition to the geometric arrangement, the capacitance of a capacitor can be influenced by inserting a suitable non-conductive material between its electrodes.

[3]The attentive reader probably did not miss the fact that the total charge of the capacitor is zero, because $Q + (-Q) = 0$. In this case, the term "charge of the capacitor" means the absolute value of the charge on one of the plates.

1.2.4 Dielectric materials

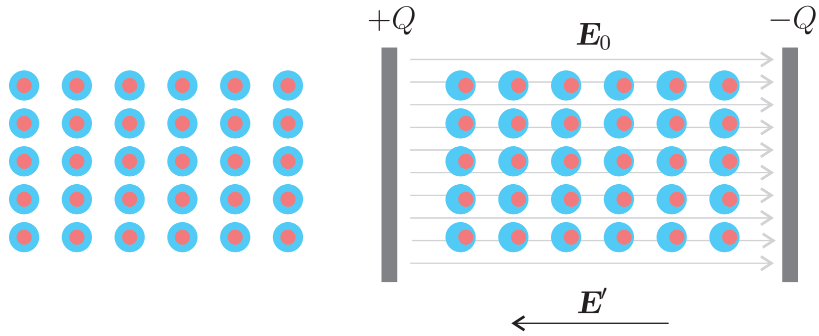
The dielectric is a non-conducting material[4], which has the ability to get polarized. In terms of atomic or molecular structure, we can divide the dielectrics into two categories.

Polar dielectrics



Molecules of polar dielectrics are characterized by the fact that even in the absence of an external electric field, they have a non-zero electric dipole moment. That is, even though the molecule is electrically neutral, the positive and negative charges are distributed asymmetrically in the molecule, it is "more positively" at one end and "more negatively" charged at the other end. A typical representative of a polar dielectric is water. Without the presence of an external electric field, the individual molecules are oriented completely randomly. By applying an external electric field with the intensity $\boldsymbol{E}_0$, they are rotated in the direction of the external field. Because the molecules perform a constant thermal motion, they are not completely oriented. The stronger the external electric field and the lower the temperature, the stronger and more complete the orientation is. The molecules arranged in the dielectric create an additional electric field $\boldsymbol{E}'$, which in an isotropic dielectrics points in the opposite direction to the external field. For the total electric field in the dielectric then $\boldsymbol{E} = \boldsymbol{E}_0 + \boldsymbol{E}'$.

Non-polar dielectrics



Whether or not they have their own non-zero dipole moment, the atoms and molecules of the dielectric in the external electric field acquire an induced dipole moment. This is caused by shifting the center of the positive charge region in the direction of the external electric field and the center of the negative charge region in the opposite direction. This again creates an additional electric field in the dielectric with the intensity $\boldsymbol{E}'$, which points in the opposite direction than the external field with the intensity $\boldsymbol{E}_0$ and thus again influences the total electric field $\boldsymbol{E} = \boldsymbol{E}_0 + \boldsymbol{E}'$. The effect of the induced dipole moments is significantly weaker compared to the effect of their own dipole moments. A typical representatives of the non-polar dielectrics are inert gases, or for example, H_2, O_2.

[4] Due to the material structure, free charge carriers cannot move at too large distances under the influence of an electric field.

1.2.5 Capacitor with a dielectric

If we insert a dielectric between the electrodes of a capacitor, the dielectric will polarize after the capacitor is charged. Due to the polarization, the magnitude of the total electric field is not E_0, but $E = E_0 - E'$. For the dielectrics, we introduce the relative permittivity ε_r as a fraction

$$\varepsilon_\mathrm{r} = \frac{E_0}{E} = \frac{E_0}{E_0 - E'},$$

which states how many times the magnitude of the electric field decreases with the presence of a dielectric compared to the case of the materialless environment (vacuum).

If we calculate the voltage between the capacitor plates (1.9) with a dielectric (where we substitute E/ε_r instead of E), we obtain

$$U = \frac{d}{\varepsilon_0 \varepsilon_\mathrm{r} S} Q.$$

For the capacitance it holds

$$C = \frac{\varepsilon_0 \varepsilon_\mathrm{r} S}{d}. \tag{1.12}$$

The quantity $\varepsilon = \varepsilon_0 \varepsilon_\mathrm{r}$ is called the absolute permittivity of a dielectric.

For many dielectrics and not very strong electric fields, the magnitude of the induced electric field E' is directly proportional to the magnitude of the intensity E_0 and thus the relative permittivity is their material constant[5]. By comparing the formulas (1.11) and (1.12) we can see that

$$C = \varepsilon_\mathrm{r} C_\mathrm{vak}. \tag{1.13}$$

Inserting a dielectric between the capacitor plates increases its capacitance ε_r-times.

Material	ε_r [–]
Vacuum	(exactly) 1
Air	1.000 54
Polystyrene	2.3–2.5
Teflon	2–2.2
Plexiglass	2.8–3.2
PVC	3.4–4
Glass	4–8
Mica	6–7
Silicone	11.7
Water (20 °C)	80.4

Table 1.1: Relative permittivity of some materials.

1.3 Experiment

1.3.1 Experimental set-up

The experimental set-up is shown in Fig. 1.1. The relative permittivity is determined by inserting the measured dielectric sample into the measuring plate capacitor $\boxed{4}$, measuring its capacity, then measuring the capacity without the sample, and using the relation (1.13)[6]. The capacitance of the measuring capacitor (with or without the sample) can be determined by measuring the linear dependence between the accumulated charge and the voltage at the electrodes, the capacitance is the direction of the dependence $Q = Q(U)$, where the voltage U is set on the high-voltage power supply $\boxed{5}$. The principle of the accumulated charge determination is schematically shown in Fig. 1.2.

[5] In the case of alternating electric fields, it must be taken into account that the relative permittivity is also a function of frequency.

[6] The relative permittivity of the air (see Table 1.1) is close to one, the replacement of the materialless environment by air in this experiment does not cause any measurable systematic error.

Figure 1.1: Experimental set-up: $\boxed{1}$ – voltmeter, $\boxed{2}$ – reference capacitor, $\boxed{3}$ – universal measuring amplifier, $\boxed{4}$ – measuring plate capacitor (with inserted dielectric sample), $\boxed{5}$ – high voltage power supply, $\boxed{6}$ – high-value protective resistor.

First, a measuring capacitor of unknown capacitance C_x is charged via a protective resistor with a resistance of 10 MΩ to the required voltage U (set on the power supply). This will accumulate a (yet unknown) charge on it

$$Q = C_x U. \tag{1.14}$$

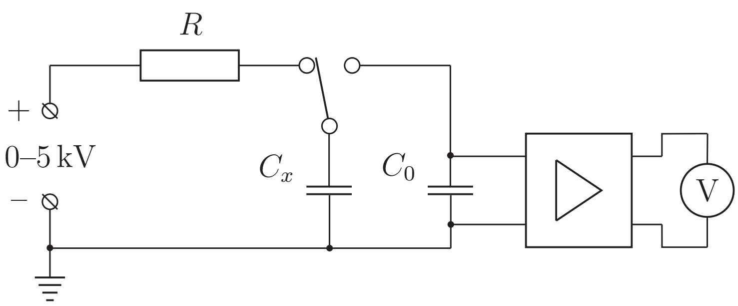


Figure 1.2: Experimental set-up.

Then, by switching the switch, the measured capacitor is disconnected from the high-voltage power supply and connected in parallel to a reference capacitor with known capacitance of $C_0 = 216\,\mathrm{nF}$. This redistributes the charge Q between the two capacitors, so that there will be a voltage U_x on them, for which it holds

$$Q = (C_0 + C_x)U_x. \tag{1.15}$$

The voltage U_x is measured with a voltmeter connected via a measuring amplifier with a high input resistance (so that the capacitors do not discharge too quickly). By combining the formulas (1.14) and (1.15) we get the relationship between the charge Q and the measured voltage U_x in the form[7]

$$Q = C_0 \frac{U U_x}{U - U_x}, \tag{1.16}$$

if $U_x \ll U$ it can be simplified as

$$Q \approx C_0 U_x. \tag{1.17}$$

The unknown capacitance C_x can be then determined employing the method of least squares[8] by approximating the measured dependence $Q(U)$ by a straight line.

[7]The perceptive reader will surely think of combining relations (1.14) and (1.15) it is possible to eliminate the charge Q and directly calculate the capacity C_x. This is true, but our intention here is to verify that there is a direct proportion between the set voltage U and the measured charge Q (relation (1.14)).

[8]For example, it is implemented at server *https://planck.fel.cvut.cz/praktikum/* — *"An universal tool for plotting graphs - least squares method"*.

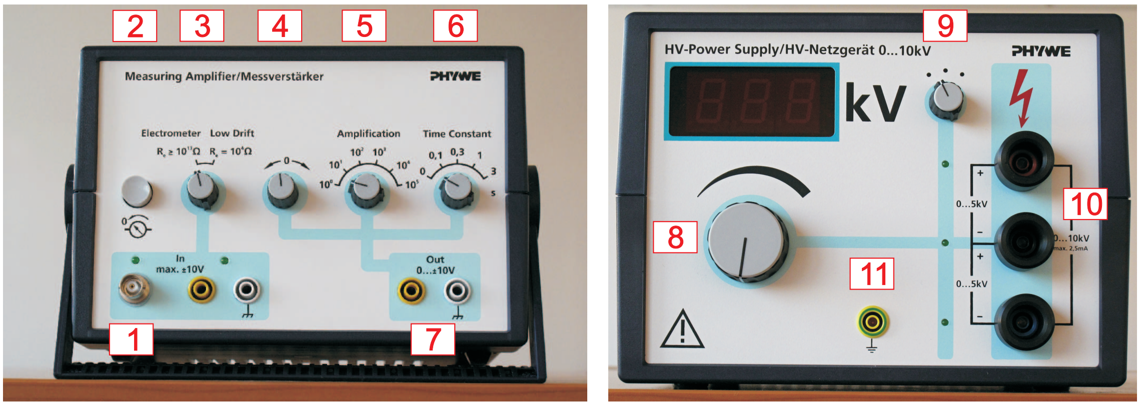


Figure 1.3: Universal measuring amplifier (left panel), high-voltage power supply (right panel), $\boxed{1}$ – high-impedance input ($R = 10\,\mathrm{T\Omega}$, input range $\pm 10\,\mathrm{V}$), $\boxed{2}$ – button for the discharging the reference capacitor, $\boxed{3}$ – input mode switch (it should be switched to position "Electrometer"), $\boxed{4}$ – potentiometer for zeroing the output voltage, $\boxed{5}$ – amplifier gain selector, $\boxed{6}$ – time-constant of the low-pass filter (it should be switched to position "0"), $\boxed{7}$ – output terminal for connecting a voltmeter (max. output voltage $\pm 10\,\mathrm{V}$), $\boxed{8}$ – potentiometer for setting the output voltage , $\boxed{9}$ – switch for the mode selection (it should be switched to the left position – upper branch, 0–5 kV), $\boxed{10}$ – output terminal 0–5 kV, $\boxed{11}$ – earth terminal.

1.3.2 Measurement safety

The experiment is switched on and off by the teacher. Do not change the wiring of the experiment, nor disconnect it. When measuring, we work with a voltage of up to 5 kV. When measuring, a protective resistor of $10\,\mathrm{M\Omega}$ must be connected in the live output of the high-voltage power supply, which limits the maximum output current to 0.5 mA. The standard[9]sets the maximum output current of the power supply to 10 mA or 3 mA, if it is necessary to touch the device with your hands during operation.

Before handling the measuring capacitor (dielectric sample replacement), set the output voltage to zero. During the measurement, a charge of the order of units of μC will accumulate on the measuring capacitor. The standard stipulates that the accumulated charge between currently accessible parts of the equipment must not exceed $50\,\mu\mathrm{C}$.

It follows from the above that there is no danger when performing the experiment according to the instructions.

Note

The dielectric strength of the air is about $3\,\mathrm{MV\,m^{-1}}$, if you bring the condenser plates too close to each other, electrical breakdown and sparking may occur. This does not damage the device.

1.3.3 Procedure

1. Ask the teacher to switch on the experiment (he/she will first check whether the high-voltage power supply is set to the minimum output voltage, it is connected to the measuring capacitor, whether it is not short-circuited and whether the reference capacitor is connected to the measuring amplifier).

[9]This is the standard ČSN 33 2000-4-41.

2. Set the output voltage of the high-voltage power supply to zero, insert a dielectric sample between the plates of the measuring capacitor and use a plastic rotating element to set the distance between the plates so that there is no air gap between the sample and the plates.

3. Use the switch on one of the capacitor plates to connect the capacitor to the high-voltage power supply. Use the potentiometer to increase the voltage by 500 V.

4. Discharge the reference capacitor with the button on the measuring amplifier. The voltmeter should show zero voltage, if not, adjust it with the appropriate potentiometer on the amplifier.

5. Connect the measuring capacitor to the reference capacitor with the switch on the measuring capacitor plate, read the voltage U_x on the voltmeter. If the voltmeter shows a voltage greater than[10] 10 V, or too low voltage, adjust the gain of the measuring amplifier, or the range of the voltmeter and repeat the measurement.

6. Proceed with step 3 up to the voltage of 5 kV.

7. Set the voltage of the high-voltage power supply to zero, loosen the plates of the measuring capacitor, remove the dielectric sample and return the plates to their original separation distance.

8. Repeat the whole measurement without a dielectric sample.

9. Plot both dependencies $Q(U)$ in one graph, calculate the relative permittivity of the dielectric sample (and its uncertainty).

10. If you have time, perform the measurements for another dielectric sample.

11. Set the voltage of the high-voltage power supply to zero, remove the dielectric sample from the measuring capacitor and ask the teacher to check and switch off the experiment.

1.4 References

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[10] The maximum output voltage of the measuring amplifier is just 10 V.