

Laboratory experiment

Determination of the Planck constant employing VA-characteristics of a LED

1.1 Tasks

1. Measure the VA characteristics of three different light-emitting diodes (LEDs) and plot the measured data in one graph.
2. From the measurements on all LEDs, calculate the value of the Planck constant.

1.2 Note

On 20 May 2019, a new definition of the SI system of units came into force. Along with this new definition, some physical constants have been fixed, including the Planck constant. The Planck constant now has the value

$$h = 6.626\,070\,15 \cdot 10^{-34} \text{ J s.} \quad (1.1)$$

This value is by definition exact, and the Planck constant need not be measured. So the actual goal of this lab experiment is not to measure the value of the Planck constant, but to test that the physical theory works, to learn some experimental techniques, and to try to process the measured data.

1.3 Theory

1.3.1 Electrons in electron clouds of isolated atoms

The state of an electron in the atomic electron cloud is completely determined by four quantum numbers n, l, m_l , and m_s . The principal quantum number n takes integer values of $1, 2, 3, \dots$, and is mainly related to the energy of the electron¹. The orbital quantum number l takes the values $0, 1, 2, \dots, n - 1$, it quantizes the magnitude of the orbital angular momentum of the electron. The magnetic quantum number m_l quantizes the projection of the orbital angular momentum of the

¹In a hydrogen atom or a hydrogen-like ion, the energy of the (single) electron depends only on this number, as n increases the negative energy of the electron increases (towards zero, the electron is bound in the atom, so the higher its energy, the weaker it is bound). In atoms with multiple electrons the situation is more complex, since in addition to the attractive interaction with the nucleus, the electrons also interact repulsively with each other, which can result in partial shielding of the charge of the atomic nucleus and the resulting non-trivial dependence of the energy of the electron on the quantum numbers n and l .

electron onto any chosen axis and it takes the values $0, \pm 1, \pm 2, \dots, \pm l$. The spin quantum number m_s takes the values $\pm 1/2$ and quantizes the projection of the internal angular momentum (spin) of the electron onto any chosen axis.

All states with the same value of n form the so-called shell, all states with the same value of n and l form the so-called sub-shell. Letters are often used instead of numbers to denote sub-shells: $s(l = 0), p(l = 1), d(l = 2), f(l = 3), \dots$. The energies of electrons in the shells of isolated atoms can only take discrete values. The energy of an electron in a sub-shell depends mainly on the quantum number n and to a lesser extent on the quantum number l . Within a given shell, the energy of the electron increases with increasing l .

According to Pauli exclusion principle, no two electrons in the same atom can have the same set of quantum number values. Hence, in one completely filled sub-shell there are $2(2l + 1)$ electrons (of different states) and in one filled shell there are $\sum_{l=0}^{n-1} 2(2l + 1) = 2n^2$ electrons. The electrons in the atom fill the individual shells and sub-shells so that the resulting energy of the atom is the smallest possible.

So, for example, neon has 10 electrons and the electron configuration of $1s^2, 2s^2, 2p^6$. This means that there are two electrons in the first ($n = 1$) shell in the s sub-shell ($l = 0$) (they differ in the spin quantum number), in the second shell ($n = 2$) in the sub-shell s there are again two electrons differing in spin number and in the sub-shell p ($l = 1$) there are 6 electrons (differing in the magnetic number $m_l = -1, 0, 1$ and the spin number $m_s = -1/2, 1/2$). Thus, all sub-shells are completely filled. Neon therefore has no weakly bound so-called valence electrons to form chemical bonds with other atoms and it is thus chemically neutral.

For example, sodium with 11 electrons has the electron configuration of $1s^2 2s^2 2p^6 3s^1$, which means that 10 electrons fully occupy shells 1 and 2 and that there is only one electron in the third shell ($n = 3$) in the s sub-shell. This valence electron is only very weakly bound to the atom, and therefore sodium easily reacts chemically with atoms that have an unoccupied state (“hole”) that can be “filled” by this electron.

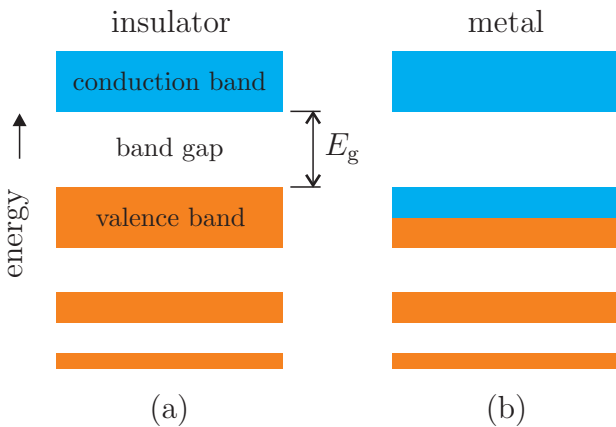


Figure 1.1: Energy bands of (a) insulators and (b) metals.

their mutual distances taking values on the order of 10^{-10} m. Because of these small distances (comparable to the dimensions of the individual isolated atoms), electron interactions also occur with neighboring atoms, resulting in the splitting of the individual discrete energy levels of the isolated atoms into as many levels as there are atoms in the crystal lattice.

Since the widths ΔE_i of the resulting *energy bands* take values on the order of electron-volts²

²One electron-volt (1 eV) is the energy gained by an electron in a potential difference of one volt. Thus, $1 \text{ eV} =$

For example, chlorine with 17 electrons has the electron configuration of $1s^2, 2s^2, 2p^6, 3s^2, 3p^5$, which means that the $1s, 2s, 2p$ and $3s$ sub-shells are completely filled. Since the $3p$ sub-shell ($l = 1$) can be filled with $2(2l + 1) = 6$ electrons, it is just one electron short of being fully occupied in chlorine. Thus, chlorine readily reacts chemically with atoms that have a valence electron that can fill this hole. For this reason, sodium chloride, for example, is a very stable compound.

1.3.2 Conduction of electric current in crystalline solids

In solids with a crystalline structure, the individual atoms are arranged in a periodic crystal lattice, with

and solids contain $N \approx 10^{23}$ atoms/cm³, the individual energy levels within a band are very tightly packed, there are very many of them and the bands can be considered continuous. Between these bands there are regions of energy that no electron can acquire. These regions are called the *band gaps*. For a solid to conduct an electric current, some electrons must move to higher energy levels.

Insulators

If the highest energy band containing electrons (the so-called *valence band*) is fully occupied, see Fig. 1.1a, the Pauli exclusion principle prevents electrons from moving to already occupied levels. Unoccupied energy levels are only available in the band above the band gap (the so-called *conduction band*), but to reach them the electron must overcome the energy E_g . If the band gap is wide enough, electrons have more or less no chance to overcome it and the material conducts almost no electric current. These materials are called *insulators*, (e.g., for diamond $E_g = 5.5$ eV, the mean kinetic energy of thermal motion of the particle at room temperature $E_k \approx 0.04$ eV).

Metals

It is characteristic of *metals* that the highest electron-occupied level is near the centre of the energy band, see Fig. 1.1b. If a voltage is applied to the metal, electrons from the highest occupied levels can gain some of the field energy to move to higher unoccupied energy levels. These so-called conduction electrons can then move freely, making metals good conductors of electric current.

Semiconductors

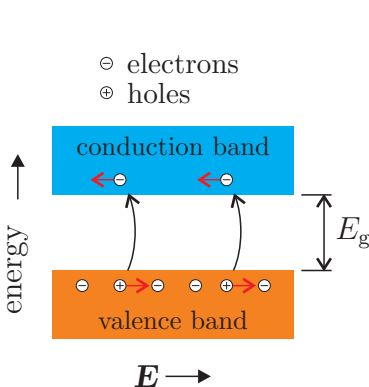


Figure 1.2: Motion of electrons and holes in electric field \mathbf{E} .

So-called *semiconductors* have a similar band structure as insulators, see Fig. 1.1a, but in the case of semiconductors the band gap between the valence and conduction band is much narrower. For example, for silicon $E_g = 1.1$ eV. Thus, for silicon (unlike diamond), even at room temperature, electrons can jump into the conduction band due to thermal agitation³. After these electrons, vacant energy states remain in the valence band—the so-called *holes*.

If we create in a semiconductor an electric field \mathbf{E} , see Fig. 1.2, electrons tend to move in the direction opposite to it⁴. Electrons in the conduction band can move freely due to the large number of unoccupied states, while electrons in the valence band can only fill unoccupied states, i.e., the holes. By this mechanism the holes drift in the direction of the electric field, their motion appears as that of positively-charged particles. Thus, in semiconductors, in addition to the electron conductivity, the hole conductivity is equally important.

Doped semiconductors

The physical properties of semiconductors can be fundamentally altered by adding small amounts of impurity atoms – by so-called *doping*.

³ 1.602×10^{-19} J.

³Resistivity of silicon is about $10^{11} \times$ higher than the one of copper at room temperature because the concentration of free charge carriers is about $10^{13} \times$ lower.

⁴For the electrostatic force acting on a particle with charge q , $\mathbf{F} = q\mathbf{E}$. Since the electron has a negative charge, the vectors \mathbf{E} and \mathbf{F} have opposite directions.

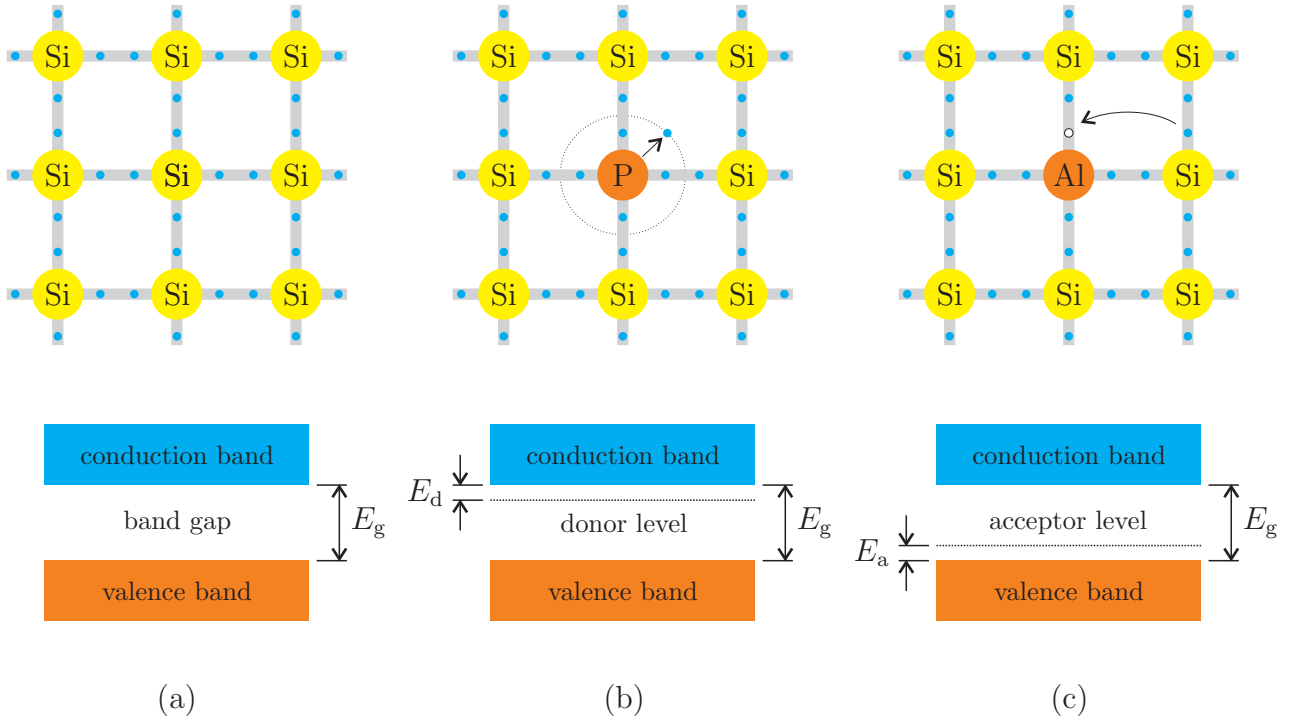


Figure 1.3: Two-dimensional model of crystal lattice and band structure of a) pure semiconductor, b) n -type semiconductor, and c) p -type semiconductor.

Figure 1.3a) shows a two-dimensional model of the crystal lattice of a typical semiconductor—silicon. The fourteen electrons in a silicon atom have the configuration $1s^2, 2s^2, 2p^6, 3s^2, 3p^2$. Each silicon atom contributes four weakly bound electrons from the $3s^2$ and $3p^2$ sub-shells to form a two-electron covalent bond⁵ with each of their four neighbors. These electrons form the valence band in silicon. If one of these bonding electrons breaks off, it can travel through the crystal lattice (we say it has jumped the band gap to the conduction band).

If we replace a silicon atom in the crystal lattice with a phosphorus atom, see Fig. 1.3b, with 15 electrons in the configuration $1s^2 2s^2 2p^6 3s^2 3p^3$, only four of the five valence electrons (2 in the $3s$ sub-shell and 3 in the $3p$ sub-shell) participate in covalent bonds with neighboring silicon atoms. The remaining electron does not participate in the bonds and is only very weakly bound to the phosphorus atom. In the band diagram [Fig. 1.3b bottom] this electron occupies a localized state near the bottom of the conduction band, the energy E_d required for its transition to the conduction band is very small compared to the width of the band gap E_g ($E_d \approx 0.05 \text{ eV}$). We call the phosphorus atom the *donor* because it easily “donates” an electron into the conduction band. Due to the low value of E_d , even at room temperature, due to thermal agitation, almost all of the electrons supplied by the phosphorus atoms are in the conduction band, thus substantially increasing the number of conduction electrons. Semiconductors doped with donor atoms are called n -type semiconductors, where n -negative expresses that the number of negative charge carriers (electrons) in the conduction band greatly exceeds the number of positive charge carriers (holes) in the valence band. In a n -type semiconductor, electrons are *majority* carriers and holes are *minority* carriers.

If we replace a silicon atom in the crystal lattice with, for example, an aluminum atom, see Fig. 1.3c, with 13 electrons in the configuration $1s^2 2s^2 2p^6 3s^2 3p^1$, only three valence electrons (2 from the $3s$ sub-shell and 1 from the $3p$ sub-shell) can participate in covalent bonds with neighboring

⁵In a covalent bond, two atoms share a pair of their electrons.

silicon atoms. Thus, in one bond between aluminum and silicon there is a missing electron, i.e., a hole. By supplying only a small amount of energy, an electron can be pulled from an adjacent bond to fill this hole, but this creates a hole in the other bond and so it moves through the crystal lattice.

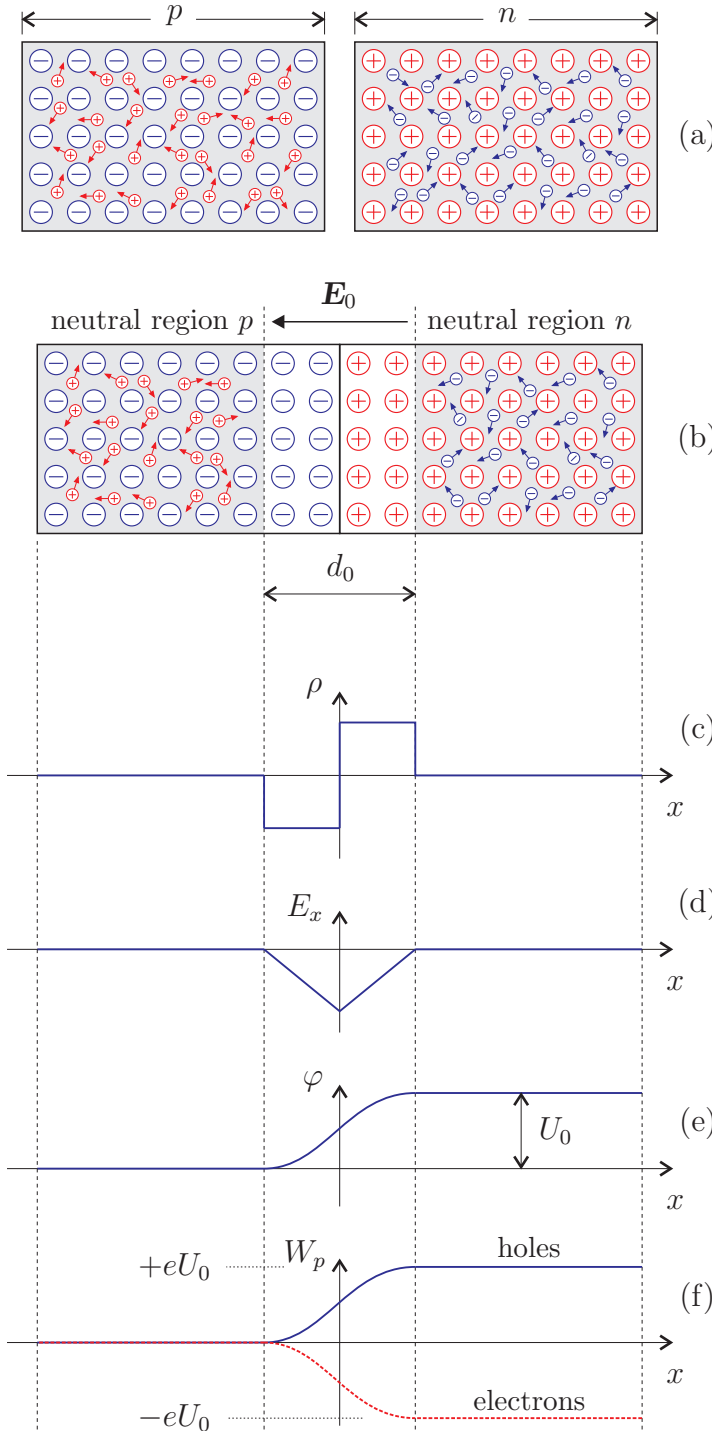


Figure 1.4: A p - n junction.

donor ions on the n -side and negatively charged immobile acceptor ions on the p -side.

This creates a region of space charge on each side of the junction plane. The volume density of the total charge ρ is schematically shown in Fig. 1.4c. This region is also called the *depletion zone*

In the band diagram [Fig. 1.3c bottom], this electron occupies a localized state near the top of the valence band, the energy E_a required to move to this level from the valence band is very small compared to the width of the band gap E_g ($E_a \approx 0.05$ eV). We call the aluminum atom an *acceptor* because it easily “accepts” an electron from the valence band. Due to the low value of E_a , even at room temperature, due to thermal agitation, almost all the acceptor levels of aluminum are occupied by atoms from the valence band, thus substantially increasing the number of holes in the valence band. Semiconductors doped with acceptor atoms are called p -type semiconductors, where p -positive indicates that the number of positive charge carriers (holes) in the valence band greatly exceeds the number of negative charge carriers (electrons) in the conduction band. In a p -type semiconductor, holes are *majority* carriers and electrons are *minority* carriers.

1.3.3 A p - n junction

A p - n junction is formed by a single crystal which is selectively doped with a p material in one part and a n material in the other, see Fig. 1.4. There is a high concentration of conduction electrons in the n -type semiconductor, while there is a high concentration of conduction holes in the p -type semiconductor. By combining the two semiconductors, see Fig. 1.4b, the concentration gradient near the junction plane results in the diffusion of conduction electrons from the n -region into the p -region and conduction holes from the p -region into the n -region. The diffusing electrons and holes near the junction plane merge (recombine) with each other, leaving positively charged immobile

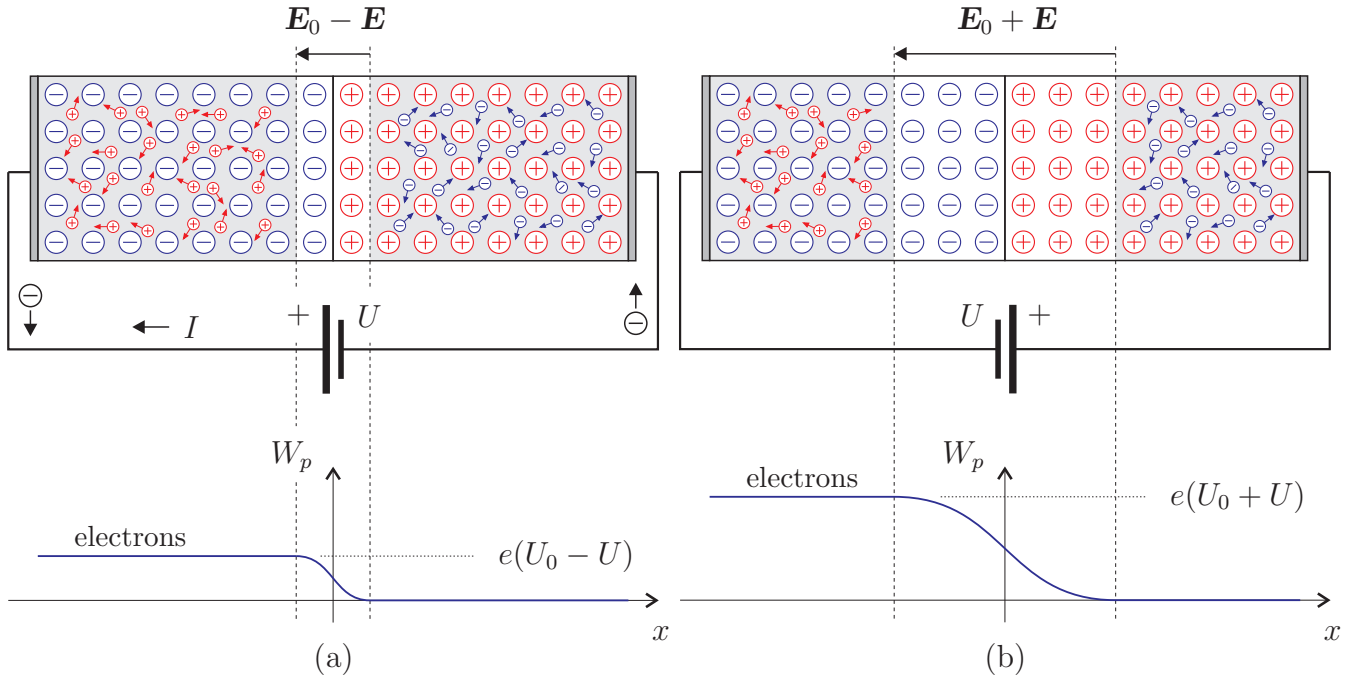


Figure 1.5: Forward- and reverse-biased p - n junction.

because it contains almost no mobile charge carriers, its width d_0 is depicted in Fig. 1.4b.

The space charge across the depletion zone gives rise to an electric field with intensity \mathbf{E}_0 , see Fig. 1.4d and so-called *contact potential* U_0 , see Fig. 1.4e. Due to the contact potential in the depletion zone, the potential energy of holes moving from the p -region to the n -region and electrons from the n -region to the p -region increases, see Fig. 1.4f, preventing further diffusion of the majority carriers across the junction plane.

For the minority carriers, i.e., holes in the n -region and electrons in the p -region, the situation is exactly the opposite, the contact potential drifts them across the junction plane. If no external voltage is applied to the p - n junction, a state of dynamic equilibrium establishes where the total current across the p - n junction generated by the motion of the majority and minority carriers is zero. Thus, the generation of the contact potential and the current of the minority carriers prevents unlimited charge transfer across the junction plane.

1.3.4 Forward- and reverse-biased p - n junction

Figure 1.5 a) shows a source of voltage U connected to a p - n junction such that the positive pole is connected to the p -region and the negative pole to the n -region. This makes the p -region more positive than before the source was connected and the n -region more negative, so electrons diffusing from the n -region to the p -region do not have to overcome the potential barrier of height eU_0 , but only $e(U_0 - U)$. This greatly increases the probability of their passage through the junction plane. The same applies for holes diffusing from the p -region into the n -region. The external voltage also results in a narrowing of the depletion zone. Since it contains almost no mobile charge carriers and thus has a high resistivity, its narrowing results in a reduction of its electric resistance.

Electrons are added to the n -region from the negative terminal of the source, electrons leaving the p -region to the external circuit can be seen as a source of holes for the p -region, which means that the semiconductor structure is constantly being replenished with majority charge carriers. The above results in a large current flowing through the p - n junction and we speak of the *forward-biased* p - n junction. The minority-carrier current is quite negligible compared to the majority-carrier

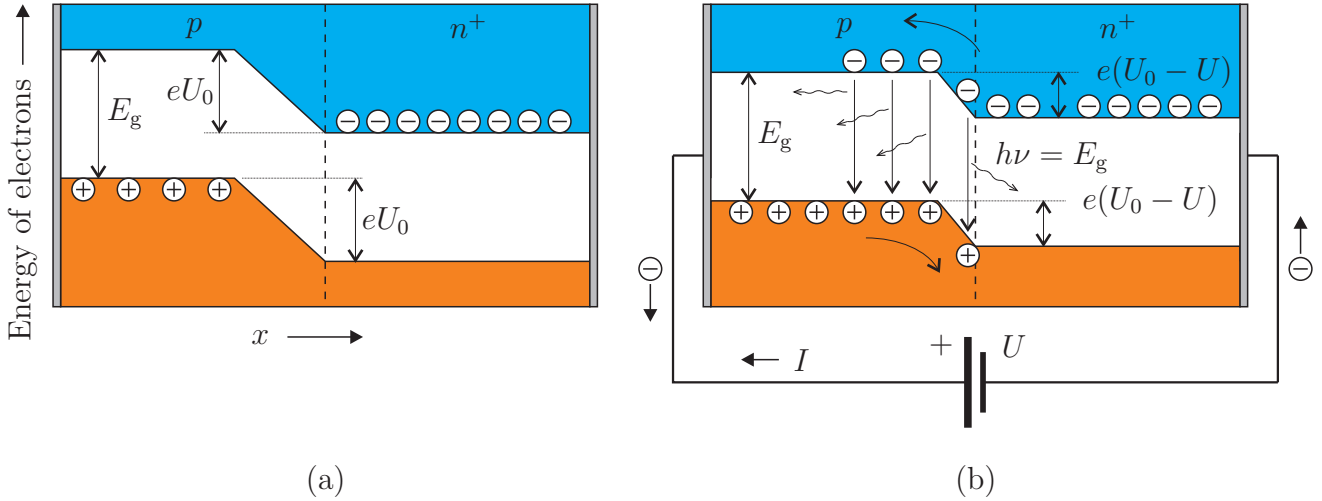


Figure 1.6: The principle of LEDs. Adapted from [3] with kind permission of the author.

current in this case, and the minority carriers are not replenished from the external source.

Figure 1.5 b) shows the opposite case, where the positive pole of the source is connected to the n -region and the negative pole to the p -region. Now the majority carriers diffusing across the junction plane must overcome a potential barrier of height $e(U_0 + U)$, which greatly reduces the probability of their passing through the junction plane. The electric resistance of the depletion zone increases due to its broadening. The current through the p - n junction is mainly due to the diffusion of minority charge carriers and is very small compared to the previous case, we speak of the *reverse-biased* p - n junction.

The above-mentioned properties of the p - n junction can be used to rectify alternating current, the corresponding electronic component is called a diode and is composed of just one p - n junction.

1.3.5 Light-emitting diode

A light-emitting diode, or LED, consists of a p - n junction in which pairs of electrons and holes recombine (by the transition of an electron from near the bottom of the conduction band to near the top of the valence band), see Fig. 1.6, with the emission of photons of light. For the emitted photons, it approximately holds⁶

$$h\nu \approx E_g \quad \Rightarrow \quad \lambda \approx \frac{hc}{E_g}, \quad (1.2)$$

where ν and λ are the frequency and wavelength of the emitted light, h is the Planck constant, and c is the speed of light. The wavelength of the emitted light is therefore inversely proportional to the width of the band gap. The emission of light occurs during the recombination of electrons and holes only in some semiconducting materials; in most cases, the energy released during the recombination is converted to the energy of the crystal lattice vibrations.

In a typical LED, the p -region is usually very narrow and the n -region is very heavily doped (n^+), so that most of the depletion zone is in the p -region, and this is also where electrons and

⁶The relation (1.2) would hold exactly if the electron were exactly at the bottom of the conduction band and the hole exactly at the top of the valence band. The energies of the electrons and holes are statistically distributed, with most of them located *close to* the edges of the energy bands, and most photons are emitted at energies for which $E_g + k_B T$ approximately holds, where k_B is the Boltzmann constant and T is the thermodynamic temperature. For the half-width of the emitted spectral line, it roughly applies that $\Delta\nu \approx 3k_B T/h$.

holes recombination and light emission occurs, reducing the probability of photon re-absorption and increasing the probability of photon emission out of the LED.

Intense light emission occurs when the p - n junction of the LED is forward-biased and a large current passes through it. Under these conditions, the depletion zone is very narrow and a large number of electron recombinations (added to the n -region from the negative pole of the source) and holes (added to the p -region from the positive pole of the source) occur.

1.3.6 VA characteristics of a p - n junction

It can be shown that for the VA characteristics of a p - n junction it applies

$$I = I_s \left[\exp \left(\frac{eU}{\eta k_B T} \right) - 1 \right] \approx I_s \exp \left(\frac{eU}{\eta k_B T} \right), \quad (1.3)$$

where I is the current through the p - n junction, U is the corresponding voltage, e is the magnitude of the elementary charge, k_B is the Boltzmann constant, T is the thermodynamic temperature (in kelvins), η is the so-called identity factor (it typically varies from 1 to 2), and I_s is the so-called reverse bias saturation current. The approximation on the right-hand side of Eq. (1.3) is valid for $I \gg I_s$. Its magnitude depends on the temperature and the band gap width E_g , where this proportionality applies

$$I_s \sim A \exp \left(-\frac{E_g}{\eta k_B T} \right), \quad (1.4)$$

where A is the contact surface area of the p - n junction.

1.4 Experiment

1.4.1 Calculation of the Planck constant

We can calculate the Planck constant if we have several LEDs that have the same contact surface area of the p - n junction and we know the wavelengths of their light. If the same current I_0 flows through all the diodes under the same conditions, the formulas (1.3) and (1.4) result in

$$const = \exp \left(\frac{eU - E_g}{\eta k T} \right) \Rightarrow eU - E_g = const.$$

If we substitute the relationship between the frequency of the emitted light and the width of the band gap (1.2), we get the formula

$$U = \frac{h}{e} \nu + const, \quad (1.5)$$

in other words, the voltage across each diode is a linear function of the frequency of the emitted light, the slope of this dependence is proportional to the Planck constant.

The measurement and calculation can therefore be done as follows. We set the same current I_0 through all LEDs. For each wavelength λ_i we measure the corresponding voltage U_i across the p - n junction. If we calculate the frequencies of the emitted light using the relation $\nu_i = c/\lambda_i$, where c is the speed of light in vacuum, we obtain pairs of values (ν_i, U_i) , which should lie on a straight line, see the relation (1.5). However, due to the presence of uncertainties and measurement errors, they lie more or less in its vicinity. An estimate of the true value of the Planck constant h is obtained by approximating the pairs of values (ν_i, U_i) by the least squares method by a straight line (first degree polynomial) of the form

$$U = a_1 \nu + a_0, \quad (1.6)$$

where the value of the absolute term a_0 is of no interest here. If we use the voltage in volts and the frequency of the emitted light in petahertz to determine the coefficient a_1 ⁷, the estimated value of the Planck constant in joule-seconds can be calculated as

$$h = 1.602 \times 10^{-34} a_1.$$

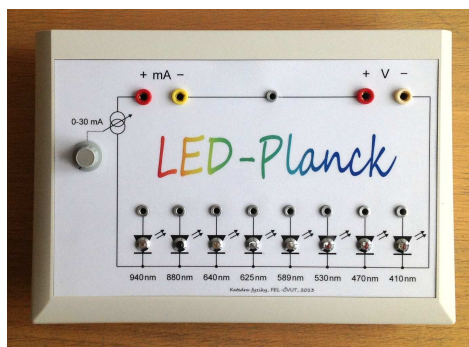
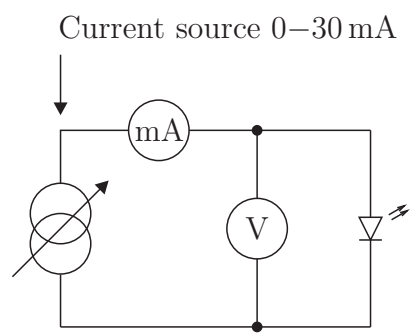


Figure 1.7: Experimental set-up.

For the calculation of the coefficient a_1 (and plotting the graphs) you can use the *Universal tool for plotting graphs*.⁸

1.4.2 Experimental set-up

The wiring of the experiment is very simple and is shown in Fig. 1.7. Using the potentiometer of the electronic current

source, the current through the LEDs can be adjusted in the range of about 0 – 30 mA.

The individual LEDs can be selected using the connecting cable, the respective wavelengths specified by the diode manufacturer are listed under the individual LEDs. Since the VA characteristic are very steep (when adjusting the current through the LEDs, the corresponding voltage changes very little), the voltmeter should always be set to the smallest usable range.

1.5 Procedure

1. Connect the experimental set-up according to Fig. 1.7.
2. Measure the VA characteristics of three LEDs in the current range 0 – 20 mA. Be sure to note their wavelengths.
3. In one graph, plot all the (three) VA characteristics⁹.
4. For some small current I_0 , e.g., 5 mA, one by one, measure the voltages across all LEDs.
5. Use the procedure described in paragraph 1.4.1 to calculate the value the Planck constant and its standard uncertainty. Plot the measured values (frequency, voltage) together with the regression line in a graph.

1.6 References

1. D. Halliday, R. Resnick, J. Walker, *Fyzika*, VUTIUM-PROMETHEUS, 2000.
2. R. A. Serway, C. J. Moses, C. A. Moyer, *Modern Physics*, Cengage Learning, 2004.

⁷It holds $1 \text{ PHz} = 10^{15} \text{ Hz}$.

⁸<https://planck.fel.cvut.cz/praktikum/>

⁹For this, you can again use the *Universal tool for plotting graphs*; in this case, approximate the measured data by exponential functions.

3. S. Kasap, *p-n Junction Devices and Light Emitting Diodes*, e-Booklet, 2001,
<http://www.kasap.usask.ca/samples/PNJunctionDevices.pdf>.

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