

Laboratory experiment

Determination of thermal conductivity of metals

1.1 Task

1. Determine the thermal conductivity of copper and aluminum alloy, compare the measured values with the corresponding values from tables.
2. Plot a graph of temperature drop along the measured samples as a function of time.

1.2 Theory

1.2.1 Energy, work, heat

Although everyone intuitively understands the meaning of the word energy, it is difficult to formulate a precise definition. We can understand energy as the ability to cause change. Energy exists in many different forms, such as mechanical, electrical, chemical, nuclear, or thermal energy. Total energy E of a thermodynamic system¹ (TS) is the sum of all its forms. Thermodynamics does not provide us with any information about the absolute value of the total energy of TS, it only deals with its changes, which is fully sufficient to solve practical problems.

The forms of contributions to the total energy of TS can be divided into two groups: macroscopic and microscopic ones. Macroscopic forms of energy are those that the system as a whole has within a given external reference frame, it is most often the kinetic energy E_k and the potential energy E_p . Microscopic forms of energy are associated with the internal atomic activity and structure of TS, they are independent of the external reference frame and their sum is called the internal energy U . For the total energy of TS we can then write

$$E = E_k + E_p + U.$$

Here we will limit ourselves only to stationary TSs, for which during the investigated process there is no change (with respect to the given reference frame) of the velocity or position of the center of gravity and therefore it applies here

$$\Delta E = \Delta U.$$

¹TS is defined as a certain amount of matter or a part of space that is the subject of investigation, which is separated from the environment by some imaginary or actual boundary.

Internal energy also has a kinetic and potential component. For example, in the case of gases, the kinetic component of the internal energy is given by the translational motion of the molecules, in the case of polyatomic molecules it is also given by their rotation and vibrations.

The mean kinetic energy of molecules is proportional to temperature, so with increasing temperature the internal energy of TS increases. Internal potential energy is related to force bonds, whether between individual molecules (latent energy), atoms within molecules (chemical energy), or even protons and neutrons in atomic nuclei (nuclear energy).

There exist only two mechanisms for energy transport to and from a closed² TS: work and heat.

The heat Q is the energy transported³ between two TSs (or the TS and its surroundings) because of a temperature difference of both the TSs (or the TS and its surroundings), where the energy transport occurs from the higher temperature location towards the lower temperature location.

The (mechanical) work W is the energy transport associated with the path action of force. If the energy transferred across the boundary of a closed TS is not heat (ie caused by a temperature difference), it is work.

Work and heat have many common features: they act across the TS boundary, they are oriented (energy is transferred to/from the TS), they are processes (the TS *has* energy but *does not have* work or heat), they are trajectory-dependent (it matters how they take place, it is not enough to know only the initial and final state of the process).

An example is the electric work

$$W_e = \int_{t_1}^{t_2} P(t)dt, \quad (1.1)$$

where $P(t) = U(t)I(t)$ is the electric power, U , I being the (time-dependent) voltage and current, or mechanical pressure-volume work

$$W_{pV} = \int_{V_1}^{V_2} p(V)dV, \quad (1.2)$$

where p is the pressure and V is the volume.

There are three different means of heat: conduction, convection, and radiation.

Conduction is the transfer of energy from the more energetic particles of matter to the less energetic ones through their mutual interactions.

It can take place in solids, liquids and gases. In fluids⁴ there are the collisions of molecules during their random (thermal) motion, in solids it is a combination of oscillations of atoms in the crystal lattice and energy transport by free electrons.

It is observed that the magnitude of the rate of thermal energy transfer (i.e., thermal energy transferred per unit time) through a thin layer of thickness Δx is proportional to temperature drop ΔT across the the layer, and its area S , and that it holds

$$\dot{Q}_{\text{cond}} = \lambda S \left| \frac{\Delta T}{\Delta x} \right|, \quad (1.3)$$

²We speak of a closed TS if there is no mass transfer between it and its surroundings (across the border).

³Regarding this definition of the term heat it is evident that, e.g., the term “heat transport” (= transport of the energy transport) does not make any sense, even if we can hear it even from a physicist. In this case, we just speak about the “transport of thermal energy,” nevertheless, the former one (and the ones similar to it) are so frequent that they are often used also in thermodynamics.

⁴That is, in liquids and gases.

Material	λ [Wm ⁻¹ K ⁻¹]
Ag (98.98%)	418
Fe (99.92%)	73
Cu (electrolytic)	395
Al (99.75%)	229
Pb	34.7
Pt	70.3
Brass	106
Bakelite	0.23
Plexiglass	0.2
Polystyrene	0.16
Water	0.63
Air	0.03

Table 1.1: Thermal conductivity of selected materials.

where the proportionality constant λ is so-called heat conductivity, which is the measure of the ability to conduct heat. Materials that are good conductors of electric current, such as copper or gold, are also good conductors of heat (the electron component of thermal conductivity is important for metals), materials such as rubber or wood are bad conductors of heat (heat is conducted here mainly through atomic oscillations in the lattice). In the limit $\Delta x \rightarrow 0$, formula (1.3) can be rewritten into the form

$$\dot{q} = \frac{\dot{Q}}{S} = \lambda \left| \frac{dT}{dx} \right|, \quad (1.4)$$

where \dot{q} is the local heat flux density magnitude, and generalized into the three-dimensional form

$$\dot{\mathbf{q}} = -\lambda \nabla T, \quad (1.5)$$

where $\dot{\mathbf{q}}$ is the local heat flux density vector; the minus sign represents the fact that the heat flows in the direction opposite to the temperature gradient (from the location with higher temperature towards the location with lower temperature). The formula (1.5) is known as the Fourier law.

Convection is the transfer of thermal energy between a solid and a fluid surrounding the solid, which involves conduction and the fluid motion. If the fluid were at rest, it would be purely the conduction. The presence of a fluid in motion facilitates the transfer of thermal energy between the solid and the fluid and greatly complicates its calculation. The agitation of the fluid may occur spontaneously (e.g., in a gravitational field—the heated fluid begins to rise due to the lower density due to the buoyancy force and is replaced by a cooler fluid), or forcibly (e.g., by a fan).

The rate of thermal energy transfer by convection is determined using the so-called Newton's law⁵

$$\dot{Q}_{\text{conv}} = -hS(T_s - T_f), \quad (1.6)$$

where h is the proportionality coefficient depending on the geometry of the body surface, the material properties of the fluid, the velocity and the nature of its flow; it is usually determined experimentally. Further, S is the surface area, T_s is the surface temperature, and T_f is the temperature of the surrounding fluid.

Radiation is the energy emitted by matter in the form of electromagnetic waves (or, from the point of view of quantum mechanics, in the form of photons) due to changes in the configuration of electrons in atomic shells.

Unlike conduction and convection, radiation allows for the energy transfer in vacuum; the material environment must be transparent for the energy transfer by radiation. The rate of the energy transfer by radiation is given by Stefan-Boltzmann law⁶

$$\dot{Q}_{\text{rad}} = -\varepsilon\sigma ST_s^4, \quad (1.7)$$

where $\sigma = 5.670\,374\,419\dots \times 10^{-8} \text{ W m}^2 \text{ K}^{-4}$ is the Stefan-Boltzmann constant, S is the body's surface area, and T_s is the body's surface thermodynamic temperature. Coefficient ε is so-called emissivity, for which it holds $0 \leq \varepsilon \leq 1$. Emissivity is a measure of the body's ability to radiate energy, generally depending on temperature and wavelength, but it is usually expressed as a single average number. In the same way that a body can emit, it can absorb incident radiation.

1.2.2 First law of thermodynamics

The first law of thermodynamics is the formulation of the law of conservation of energy, based on observation that energy during a physical process cannot be created or destroyed, it can only

⁵The minus sign in front of the formula captures the fact that the heat supplied to the TS (in this case a solid) is usually understood as positive.

⁶The minus sign here again describes the fact that the body loses energy from radiation.

change its form. For a closed (no mass transfer) and stationary (no change in velocity and position) TS, we can formulate the first law of thermodynamics as

$$\Delta E = \Delta U = Q + W, \quad (1.8)$$

thus, the increase in internal energy of TS is equal to the sum of the heat supplied and the work performed on the system, where we consider the heat supplied to be positive as well as the work performed by external forces on the system.

1.2.3 Heat conduction equation

The above considerations can be used to derive the so-called heat conduction equation. Consider an imaginary region of volume V in a solid, into/from which energy can only transfer by conduction through the surface S . It applies to the time increase of internal energy within the surface S that

$$\frac{d}{dt} \int_V \rho u dV = - \int_S \dot{\mathbf{q}} \cdot d\mathbf{S}, \quad (1.9)$$

where u is the internal energy per unit mass at a given point, and ρ is the mass density. The minus sign on the right-hand side represents the fact that the elementary vector of the surface $d\mathbf{S}$ is oriented outside the volume V , and we consider the heat supplied inside the TS as positive.

If we apply the Gauss law to the right-hand side of Eq. (1.9) and move the time-derivative on the left-hand side inside the integral (the volume V and mass density ρ are considered constant), we get

$$\int_V \rho \frac{\partial u}{\partial t} dV = - \int_V \nabla \cdot \dot{\mathbf{q}} dV \quad \Rightarrow \quad \int_V \left(\rho \frac{\partial u}{\partial t} + \nabla \cdot \dot{\mathbf{q}} \right) dV = 0.$$

The integral on the right-hand side will be identically equal to zero for any volume V if its argument is zero, that is, if it applies

$$\rho \frac{\partial u}{\partial t} = -\nabla \cdot \dot{\mathbf{q}}. \quad (1.10)$$

For solids and liquids, if there is no change of state due to the thermal energy transfer or chemical reactions, this applies

$$du = cdT, \quad (1.11)$$

where c is the so-called specific heat capacity, which is the energy which we have to supply the unit mass of the substance in order to increase its temperature by one kelvin. If we substitute the relations (1.5) and (1.11) into Eq. (1.10), we obtain the heat conduction equation in the form

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c} \nabla^2 T, \quad (1.12)$$

where we have considered the heat conductivity λ constant and it has been put outside the divergence. In the steady-state, when the temperature does not change in time, it holds $\partial T / \partial t = 0$, and Eq. (1.12) reduces into the Laplace equation

$$\nabla^2 T = 0. \quad (1.13)$$

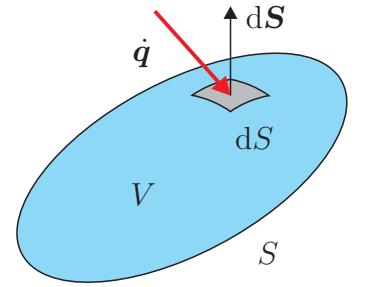


Figure 1.1: Regarding heat conduction equation.

1.3 Experiment

The arrangement of the experiment for measuring the thermal conductivity of metals is schematically depicted in Fig. 1.2. Metal rod with cross-section S is immersed on one side in a vessel with a mixture of water and ice and on the other side it is equipped with a heating element. The immersed part of the rod is kept at approximately constant temperature of $T_0 = 0^\circ\text{C}$. Due to the work of electric current in the heating element, heat $\dot{Q} = U_{\text{ef}}I_{\text{ef}}$ is supplied to the rod every second, where U_{ef} is the effective value of the voltage, and I_{ef} is the effective value of the electric current through the heating element (the end of the rod with the heating element is insulated, so that we assume that all the supplied heat passes into the rod).

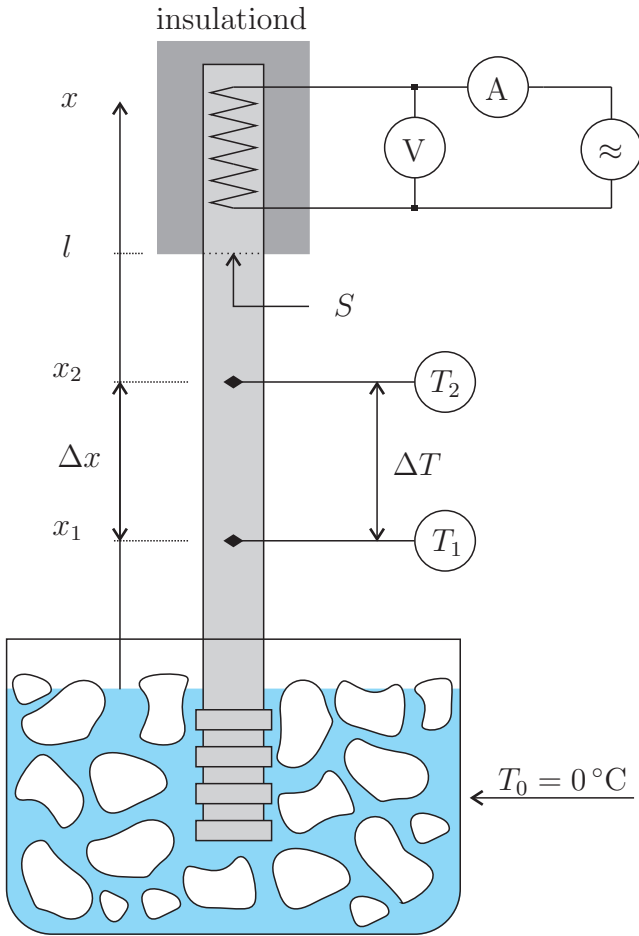


Figure 1.2: Experimental set-up.

We further assume that the rod is so short and thick that we can neglect the heat flux into the surrounding air. Additionally, we assume that the temperature in the rod is a function of only the longitudinal coordinate x along the rod (this is in accordance with the previous assumption, because with respect to Eq. (1.5) the derivative of the temperature with respect to the normal is zero on the rod's sides). From here, it follows that $T(x, y, z, t) = T(x, t)$.

After switching on the heater, the temperature along the rod starts to change (increase) due to the supplied heat. This will continue until the steady state is reached where all the heat supplied is equal to the heat transferred to the vessel with the mixture of water and ice.

In the steady state, with respect to the above mentioned and Eq. (1.13) it holds for the temperature

$$\frac{d^2T}{dx^2} = 0 \quad \Rightarrow \quad T(x) = ax + b, \quad (1.14)$$

where a and b are (integration) constantss. Near the water-ice mixture surface ($x = 0$), the rod's temperature will be approximately zero degrees Celsius, $T(x = 0) = T_0 \approx 0^\circ\text{C}$, substitution into Eq. (1.14) then yields

$$T(x) = ax + T_0. \quad (1.15)$$

At the location $x = l$ heat $\dot{Q} = U_{\text{ef}}I_{\text{ef}}$ enters every second the rod through the surface S , so that according to Eq. (1.4) it holds

$$\dot{q} = \frac{\dot{Q}}{S} = \frac{U_{\text{ef}}I_{\text{ef}}}{S} = \lambda \left(\frac{dT}{dx} \right)_{x=l}. \quad (1.16)$$

Substituting this boundary condition into Eq. (1.15) results in

$$a = \frac{U_{\text{ef}}I_{\text{ef}}}{\lambda S} \quad \Rightarrow \quad T(x) = \frac{U_{\text{ef}}I_{\text{ef}}}{\lambda S}x + T_0. \quad (1.17)$$

The heat conductivity of a given material (rod) can be determined as follows. A thermocouple-probe measuring temperature T_1 is placed at point x_1 , another one measuring temperature T_2 is positioned at point x_2 . Substituting these values into Eq. (1.17) then yields

$$T_1 = \frac{U_{\text{ef}}I_{\text{ef}}}{\lambda S}x_1 + T_0, \quad T_2 = \frac{U_{\text{ef}}I_{\text{ef}}}{\lambda S}x_2 + T_0 \quad \Rightarrow \quad T_2 - T_1 = \frac{U_{\text{ef}}I_{\text{ef}}}{\lambda S}(x_2 - x_1), \quad (1.18)$$

and thus

$$\lambda = \frac{U_{\text{ef}}I_{\text{ef}}}{S} \frac{\Delta x}{\Delta T}, \quad (1.19)$$

where Δx is the distance between the thermocouple probes, and ΔT is the corresponding temperature difference.

1.4 Procedure

1. The measurement is performed in the same way for both samples (copper and aluminum alloy).
2. Measure the distance Δx between the thermocouple probes and the diameter of the samples (rods) for the determination of the cross-sectional area S .
3. Check the experiment connection according to Fig. 1.2. Fill the glass vessel with mixture of water and ice.
4. Immerse the measured sample into the water-ice mixture (the lower thermocouple probe must be above the surface) and monitor the temperature of the mixture in the vessel, while constant stirring, until it stabilizes at value of ca. 0°C . Measure the initial temperature difference ΔT .
5. Switch on the heater (soldering iron).
6. Read the temperature difference ΔT in the interval of one or two minutes until the steady state is reached (the temperature difference ΔT stops increasing).
7. When measuring with an ammeter and voltmeter, monitor the power input of the heater and record any possible changes.
8. During the measurement **incessantly** stir the water-ice mixture and if needed, add the ice.
9. Plot both the measured time-dependences of ΔT into one graph.
10. Calculate the thermal conductivity for both samples by substituting the measured values⁷ into Eq. (1.19).

1.5 Digital thermometer Greisinger GMH 3230

It is a two-channel thermometer with thermocouple probes, the ends of which are located inside the measured samples. Be careful when potential connecting/disconnecting the probes to/from the thermometer, the wires are quite thin.

The thermometer is switched on/off by pressing the button **ON/OFF**. On the two-line display of the thermometer, temperatures T_1 , T_2 a ΔT (temperature difference) can be displayed. The

⁷Use the temperature difference ΔT at the steady state.

display mode is switched by repeatedly pressing the button **Set/Menu**, the selected display mode (for both the lines) is indicated by arrows near the symbols **T1**, **T2**, and **DIF**. Be careful with the button **Tara** which sets the temperature-difference display to zero (used to monitor temperature difference changes).

1.6 References

1. Yunus Cengel, Michael Boles: Thermodynamics: An Engineering Approach, McGraw-Hill Science/Engineering/Math, 2010.
2. Michal Bednařík, Petr Koníček, Ondřej Jiříček: Fyzika I a II – Fyzikální praktikum, [skriptum], Vydavatelství ČVUT, Praha, 2003.

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