Laboratory experiment

Measurement of coefficient of linear thermal expansion

1.1 Task

1. Determine coefficient of linear thermal expansion for at least two materials.

2. For the measured samples, plot the graph of their change in length as a function of the temperature change.

1.2 Theory

All substances, whether solids, liquids or gases, change their dimensions as the temperature changes, and in most cases these dimensions increase with increasing temperature[1]. Within this text, we will focus on solids.

1.2.1 The nature of thermal expansion in solids

The solids hold together due to the binding forces for which the electrostatic interaction of negatively charged electrons with the positively charged ions of the crystal lattice are responsible. These forces can be both attractive and repulsive, their balance means stability, thus determining the equilibrium distance between the individual atoms. The dependence of the potential energy U of the binding forces as a function of inter-atomic distance r is shown in Fig. 1.1.

The relation between the force $\boldsymbol{F}$ and the potential energy can be written as

$$\boldsymbol{F} = -\frac{\partial U}{\partial r}\boldsymbol{r}_0.$$

Figure 1.1: Binding energy of inter-atomic forces.

It is clear from here that the equilibrium position is located at the minimum of potential energy. If there is any deviation from the equilibrium position, the potential energy will always increase and

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[1]An exception is, for example, the most widespread liquid on Earth–water, whose volume in the temperature interval $0\,°\mathrm{C} - 4\,°\mathrm{C}$ decreases with increasing temperature (water anomaly).
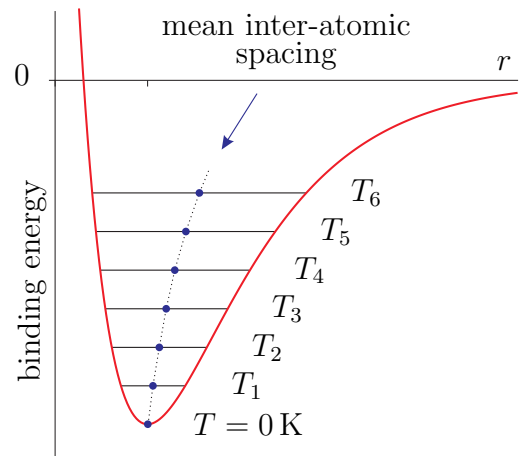
Substance	α $[10^{-6}\,°\mathrm{C}^{-1}]$	Substance	α $[10^{-6}\,°\mathrm{C}^{-1}]$
fused quartz	0.5	concrete	12
Invar[2]	1.6	copper	17
glass (ordinary)	8.5	brass	19
steel	11	aluminum	23

Table 1.1: Mean coefficient of linear thermal expansion of some substances in temperature range 0 – 100 °C.

the binding force will act against the deviation, in the direction of the equilibrium position.

The atoms in the crystal lattice are not at rest, they perform thermal oscillations, where the energy of these oscillations increases with temperature. As the potential energy function is not symmetric around the equilibrium distance, see Fig. 1.1, with increasing temperature, the maximum distance to which the atoms in the field of binding forces can deviate increases more quickly than the minimum distance, to which they can approach each other. This leads to an increase in the mean distance between the atoms in the crystal lattice with increasing temperature and thus to thermal expansion. If the potential energy function were symmetrical with respect to the equilibrium distance, thermal expansion would not occur.

1.2.2 Linear thermal expansion

Due to the phenomena described above, solids change their dimensions due to a change in temperature. The relative change in the linear dimensions of a body (for example, the length of a rod) can be expressed using the so-called coefficient of linear thermal expansion (CLTE) α', which can be defined as

$$\alpha' = \frac{1}{l_0}\frac{\mathrm{d}l}{\mathrm{d}t}, \tag{1.1}$$

where $l = l(t)$ is the length at the given temperature t and l_0 is the length at some chosen temperature, e.g., 0 °C. The unit[3] of CLTE is $°\mathrm{C}^{-1} = \mathrm{K}^{-1}$. Experiments show that the coefficient α', which is a material parameter, does not change very much with temperature, so it is possible (at least in a not very large temperature range) to consider it a constant. Integration of Eq. (1.1) then results in

$$\mathrm{d}l = l_0\alpha'\,\mathrm{d}t \quad \Rightarrow \quad \int_{l_0}^{l}\mathrm{d}x = \alpha'l_0\int_{0}^{t}\mathrm{d}\tau \quad \Rightarrow \quad l - l_0 = \alpha'l_0 t \quad \Rightarrow \quad l = l_0\left(1 + \alpha't\right). \tag{1.2}$$

As we set the lower integration limit in degrees Celsius (0 °C), we have to substitute in Eq. (1.2) for the temperature t in degrees Celsius as well. For higher temperature ranges the linear formula (1.2) needs not be accurate enough. However, it can be modified by adding a quadratic (or a cubic, ...) term into a form

$$l = l_0\left(1 + \alpha'_1 t + \alpha'_2 t^2\right). \tag{1.3}$$

For example, for copper it holds $\alpha'_1 = 1.48 \cdot 10^{-5}\,°\mathrm{C}^{-1}$, $\alpha'_2 = 1.85 \cdot 10^{-8}\,°\mathrm{C}^{-2}$. If we substituted, e.g., $t = 100\,°\mathrm{C}$ into the second and third term of Eq. (1.3) for copper, we would find out that

$$\frac{\alpha'_2 t}{\alpha'_1} = 0.125,$$

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[2]Invar is nickel-steel alloy (64 % Fe, 36 % Ni) with a very small thermal expansion.

[3]This equality follows from the fact that in the denominator of Eq. (1.1) there is a temperature difference. The Celsius and thermodynamic temperature scales are only shifted relative to each other, the magnitude of one degree Celsius and one Kelvin are the same.

so by neglecting the quadratic term would result in an error of $12.5\,\%$. Therefore, if we want to use a linear formula in a larger temperature range, it is possible to use an average value of CLTE α in this temperature range and to write

$$l_2 = l_1\left[1 + \alpha(t_2 - t_1)\right] \qquad \text{or in a short} \qquad \Delta l = \alpha l \Delta t. \tag{1.4}$$

Examples of CLTEs for some substances are given in Tab. 1.1.

1.2.3 Volumetric thermal expansion

Of course, if linear dimensions of solids change with temperature, their volume changes as well. If a body is made of a homogeneous and isotropic material, the calculation of the volumetric thermal expansion is very easy.

Let's assume that at temperature t_1, a prism has volume of $V_1 = a_1 b_1 c_1$. For its volume at temperature t_2, employing Eq. (1.4), we can write

$$V_2 = a_2 b_2 c_2 = a_1 b_1 c_1 (1 + \alpha\Delta t)^3 = V_1\left[1 + 3\alpha\Delta t + 3(\alpha\Delta t)^2 + (\alpha\Delta t)^3\right] \approx$$
$$\approx V_1\left(1 + 3\alpha\Delta t\right) = V_1\left(1 + \beta\Delta t\right) \qquad \Rightarrow \qquad \Delta V = \beta V \Delta t, \tag{1.5}$$

where $\beta = 3\alpha$ is so-called coefficient of volumetric thermal expansion (CVTE). Within the derivation of Eq. (1.5), quadratic and cubic terms were discarded, which is possible provided that $\alpha\Delta t \ll 1$, which is fulfilled if the temperature difference Δt is not too large.

Experience has shown that uniform heating does not cause mechanical stresses in a homogeneous body. It follows from here that all the layers of the body expand independently of each other and therefore the same way. Thus, if there were a cavity in the body, it would expand as if it were filled with the material that forms its walls.

1.3 Experimental set-up

Figure 1.2: Experimental set-up.

Measurements, on the basis of which it is possible to determine the CLTE, are carried out using a dilatometer, see Fig. 1.2. It consists of a heater $\boxed{2}$ equipped with a thermostat, a thermometer, a water tank $\boxed{1}$ serving as a heat reservoir, a clamping bench for fixing the measured samples $\boxed{4}$, a clock gauge $\boxed{3}$ for the measurement of the samples' lengthening, and supply hoses. The measured samples $\boxed{5}$ have the shape of hollow rods, through which hot water flows, which heats them from the inside to the required temperature.
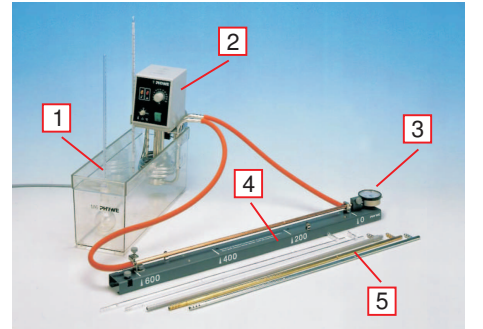
1.4 Procedure

The measurement procedure is the same for all samples. Conduct the measurement for at least two different materials.

1. Fix the measured sample (rod) in the clamping bench at the maximum possible distance of 600 mm. Attach the supply hoses to the ends of the rod and make sure that they hold tightly enough. Pour the coldest possible tap water approx. 2 cm below the edge into the water tank. Carefully attach the clock gauge to the end of the clamping bench and make sure that its measuring tip rests on the sample to be measured.

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2. Set the thermostat to the lowest possible temperature and switch it on using the switch on the front panel. Water starts to flow through the measured sample. Wait a while until the temperature equalizes and note the initial temperature t_0 and the value l_0, shown by the clock gauge. You can rotate the clock gauge sleeve with attached scale to set the value l_0 to zero. From this point on, do not touch the measured sample or the clock gauge to avoid their undesired displacement.

3. Set the thermostat to a temperature approx. $5\,°\mathrm{C}$ higher than that shown by the thermometer.

   If the water is heating up, then the orange LED on the thermostat is bright. If the water temperature approaches the pre-set one, then the heating slows down, which is indicated by the dimming of the LED. If the heating of the water has ended, the LED turns off. Read the water (and sample) temperature t_i on the thermometer and the lengthening l_i shown by the clock gauge.

4. Repeat the previous step up to the temperature of ca. $60\,°\mathrm{C}$.

5. Turn off the heater and pump and drain the water from the tank into the bucket. Carefully remove the gauge clock from the clamping bench and place it in the box. Remove the supply hoses from the measured sample **in such a way that no remaining water flows onto the table** – lift the clamping bench with the sample so that this water flows into the tank. Remove the measured sample from the clamping bench. Pour the water from the bucket into the sink.

6. If you are continuing the measurement, select another sample and continue with point 1, when you are done, wipe the work table.

1.5   Processing the measured data

For each measured sample, calculate the CLTE and its uncertainty.

You can do this by approximating the measured values ($\Delta t_i = t_i - t_0, \Delta l_i = l_i - l_0$) by a straight line (1$^{\mathrm{st}}$ degree polynomial) employing the least squares method. Then, you calculate the CLTE as

$$\alpha = \frac{A}{L},$$

where A is the straight line slope, and $L = (600 \pm 1)\,\mathrm{mm}$ is the measured sample length. Plot all the measured values ($\Delta t_i, \Delta l_i$) together with the corresponding straight lines in one graph. For the calculations as well as plotting the graphs, you can use the tool *An universal tool for plotting graphs–least squares method* at server *https://planck.fel.cvut.cz/praktikum/*.

1.6   References

1. J. B. Slavík a kol., *Základy fysiky I.*, ČSAV, Praha, 1962.

2. D. Ilkovič, *Fyzika pre študujúcich na vysokých školách technických*, SVTL, Bratislava, 1962.

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