Laboratory experiment

Measurement of the volume of rigid bodies

1.1 Tasks

1. Measure the volume of a prism or a cylinder.
2. Calculate the combined standard uncertainty of individual characteristic dimensions of the examined object.
3. Calculate the combined standard uncertainty of the volume of the examined object.

1.2 Theory

The aim of this exercise is to learn to process a simple physical experiment and evaluate the measured data. For this purpose, measuring the volume of regular objects whose characteristic dimensions are slightly deformed is very well suited. We choose for this exercise objects in the shape of a cylinder or a prism. The following applies to the respective volumes:

$$V = \frac{1}{4}\pi d^2 h = f(d, h) \qquad \text{(cylinder)} \quad \text{and} \qquad V = a\ b\ c = f(a, b, c) \qquad \text{(prism)}, \tag{1.1}$$

where d is the base diameter and h is the height of the cylinder, and a, b, c are the individual sides of the prism. The best estimate of the *true value* of the volume of these objects can be calculated as

$$\overline{V} = \frac{1}{4}\pi \overline{d}^2 \overline{h} \qquad \text{(cylinder)} \quad \text{and} \qquad \overline{V} = \overline{a}\overline{b}\overline{c} \qquad \text{(prism)}, \tag{1.2}$$

where the bar over a symbol denotes the mean value calculated from individual measured values. The combined standard uncertainty of the cylinder volume can be calculated as

$$u_\mathrm{C}(\overline{V}) = \sqrt{\frac{\pi^2 \overline{d}^4}{16} u_\mathrm{C}^2(\overline{h}) + \frac{\pi^2 \overline{d}^2 \overline{h}^2}{4} u_\mathrm{C}^2(\overline{d})}, \tag{1.3}$$

where $u_\mathrm{C}(\overline{h})$ and $u_\mathrm{C}(\overline{d})$ are the combined standard uncertainties of the cylinder height and diameter, respectively. Similarly, for the combined standard uncertainty of the prism volume we get

$$u_\mathrm{C}(\overline{V}) = \sqrt{(\overline{b}\overline{c})^2 u_\mathrm{C}^2(\overline{a}) + (\overline{a}\overline{c})^2 u_\mathrm{C}^2(\overline{b}) + (\overline{a}\overline{b})^2 u_\mathrm{C}^2(\overline{c})}, \tag{1.4}$$

where $u_\mathrm{C}(\overline{a})$, $u_\mathrm{C}(\overline{b})$, and $u_\mathrm{C}(\overline{c})$ are the combined standard uncertainties of the individual sides of the prism.

Direct measurement of the characteristic dimensions

Let the measurement of a quantity x (a characteristic dimension of the given object) be repeated n-times, where $n \gg 1$. The measured quantities make up a set $x_1, x_2, x_3, \ldots, x_n$.

According to the theory of random errors, the true value of the measured quantity x can be estimated as the mean value $\overline{x}$ of the set, which is defined as

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \ . \tag{1.5}$$

The Type A standard uncertainty of $\overline{x}$ is calculated as an estimate of the standard deviation of the quantity $\overline{x}$ which reads

$$u_{\mathrm{A}}(\overline{x}) = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \overline{x})^2}{n(n-1)}}, \tag{1.6}$$

The Type B standard uncertainty can be estimated as

$$u_{\mathrm{B}} = \frac{\Delta}{\sqrt{12}}, \tag{1.7}$$

where Δ is the size of the smallest scale division of the measuring instrument used (vernier caliper or micrometer). The combined standard uncertainty can be then calculated as

$$u_{\mathrm{C}}(\overline{x}) = \sqrt{u_{\mathrm{A}}^2(\overline{x}) + u_{\mathrm{B}}^2}. \tag{1.8}$$

1.3 Procedure

1. Repeat the measurement of each and every characteristic dimension of the given object at least $n = 10\times$. This way, you get a set of measured values $x_1, x_2, \ldots, x_n$ for each of the dimensions. For each of the characteristic dimensions, perform the individual measurements, if possible, at different positions.
2. For each of the characteristic dimensions, calculate the mean value and standard uncertainties u_{A}, u_{B}, and u_{C}.
3. Calculate the best estimate of the object's volume and its combined standard uncertainty.

1.4 References

[1] Kirkup, L., Frenkel, R. B., *An Introduction to Uncertainty in Measurement Using the GUM (Guide to the Expression of Uncertainty in Measurement)*, Cambridge University Press; 1 edition, 2006.

[2] *Guide to the Expression of Uncertainty in Measurement*, ISO, 1995.

1.5 Appendix: Vernier caliper and micrometer

1.5.1 Vernier caliper

If we measure length using a millimeter scale, we can reliably determine millimeters and we can only estimate fractions of millimeters. Therefore, for many measuring instruments, the estimate of the fraction of the division of the scale is replaced by reading on the auxiliary scale – the vernier scale.

One of the basic measuring instruments equipped with this device is the vernier caliper. The vernier scale is the auxiliary scale that can freely move along the adjacent main scale, and it is usually divided into $n = 10$ divisions (decimal vernier), which correspond to $n - 1 = 9$ divisions on the main scale. On the decimal vernier, we read tenths of a millimeter. For the decimal vernier, the zero line of the auxiliary scale indicates the number of whole divisions on the main scale, the number of tenths of the division of the main scale is given by the line of the auxiliary scale which is closest to one of the lines of the main scale.

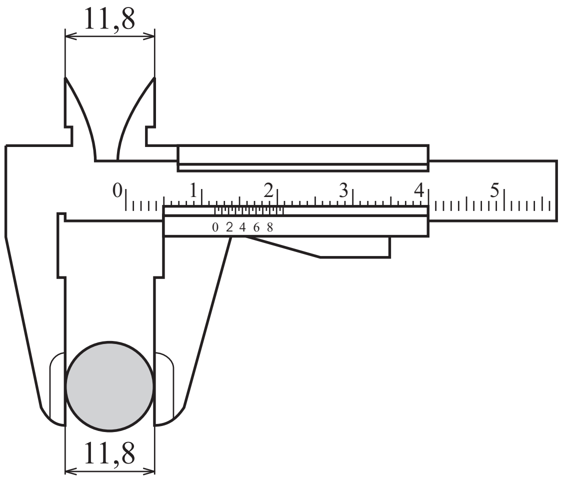

Figure 1.1: Measurement with the vernier caliper.

With the twentieth vernier one division of the main scale (1 mm) is divided into $n = 20$ divisions and we can read the length with the accuracy of 0.05 mm. Similarly, with the fiftieth vernier one division of the main scale (1 mm) is divided into $n = 50$ divisions and we can read the length with the accuracy of 0.02 mm.

1.5.2 Micrometer

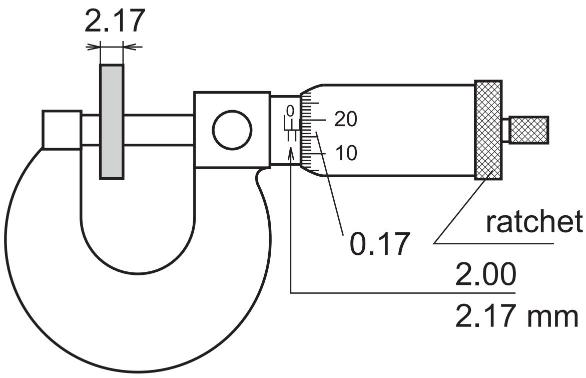

Figure 1.2: Measurement with the micrometer.

Small dimensions can be also measured with the micrometer. The micrometer consists of two main parts. The moving part resides on a precisely machined screw with 2 threads per millimeter, and thus one complete revolution moves it through a distance of 0.5 millimeter. The revolving thimble of the moving part is divided into 50 graduations. The fixed part of the micrometer is equipped with half a millimeter scale.

The reading on the micrometer is made as follows. The number of lines on the fixed scale shows the number of millimeters and half millimeters, the divisions on the revolving thimble indicate the hundredths of a millimeter. The measurement is performed by attaching the measured object to the fixed part of the micrometer (the anvil) and rotating the thimble until the movable jaw (the spindle) fits closely to the measured object. In order not to deform the measured object or the moving part by over-tightening the jaws of the micrometer, the grooved part of the thimble is separated from it by a friction clutch – the ratchet, which begins to slip as soon as the desired force is reached. This ensures that the screw is always tightened with the same force without deformation of the measured object.